EE-338
Digital Signal Processing

Filter Design Assignment

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Contents

1 Student Details

Name : Anubhav Bhatla
Roll Number : 200070008
Filter Number : 4
Method used : Cascade of bandpass and bandstop filters

Filter Number = 4 = 11Q + R
Therefore, Q = 0; R = 4
Frequency Band Group-I : 40 to 70 kHz
Frequency Band Group-II : 190 to 220 kHz

2 Bandpass Filter Details

2.1 Un-normalized Discrete-time Filter Specifications

Given below are the filter specifications for the required bandpass filter:

- Passband : 40 - 220 kHz
- Stopband : 0 - 35 kHz and 225 - 300 kHz
- Transition band : 5 kHz on either sides of the passband
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1 - \delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

2.2 Normalized Digital Filter Specifications

Sampling Rate = 600 kHz corresponds to 2π on the normalized frequency axis.

$$f_s \rightarrow 2\pi$$
$$\omega = 2\pi \times f/f_s$$

Therefore the normalized discrete filter specifications are as follows:

- Passband : $40\pi/300$ - $220\pi/300$
- Stopband : 0 - $35\pi/300$ and $225\pi/300$ - π
- Transition band : $5\pi/300$ on either sides of the passband
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1 - \delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

2.3 Analog Filter Specifications

The bilinear transformation is given as:

$$\Omega = tan(\omega/2)$$

Therefore the corresponding analog filter specifications are as follows:

- Passband : 0.2126 (Ω_{p_1}) - 2.246 (Ω_{p_2})
- Stopband : 0 - 0.1853 (Ω_{s_1}) and 2.4142 (Ω_{s_2}) - ∞
- Transition band : 0.1853 - 0.2126 and 2.246 - 2.4142
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

2.4 Frequency-transformed Lowpass Analog Filter

The bandpass transformation is given as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p_1}\Omega_{p_2}} = 0.691$$
$$B = \Omega_{p_2} - \Omega_{p_1} = 2.0334$$

The lowpass transformations for various key points are given below:

Ω	Ω_L
0^+	$-\infty$
0.1853 (Ω_{s_1})	-1.1761 ($\Omega_{L_{s_1}}$)
0.2126 (Ω_{p_1})	-1 ($\Omega_{L_{p_1}}$)
0.691 (Ω_0)	0
2.246 (Ω_{p_2})	1 ($\Omega_{L_{p_2}}$)
2.4142 (Ω_{s_2})	1.09 ($\Omega_{L_{s_2}}$)

Therefore the corresponding lowpass analog filter specifications are as follows:

- Passband Edge : 1 (Ω_{L_p})
- Stopband Edge : $\min(|\Omega_{L_{s_1}}|, |\Omega_{L_{s_2}}|) = 1.09$ (Ω_{L_s})
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

2.5 Butterworth Analog Lowpass Transfer Function

Based on the tolerance in the passband and the stopband (both equal to δ), we define two new quantities:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.4444$$

Using these newly defined quantities, the minimum order for the Butterworth filter is given as:

$$N_{min} = \lceil \frac{log(D_2/D_1)}{2log(\Omega_{L_s}/\Omega_{L_p})} \rceil = \lceil 27.4532 \rceil = 28$$

The cutoff frequency (Ω_c) of the analog lowpass analog filter has the following constraint:

$$\frac{\Omega_{L_p}}{D_1^{1/2N}} \leq \Omega_c \leq \frac{\Omega_{L_s}}{D_2^{1/2N}}$$

$$1.0172 \leq \Omega_c \leq 1.019$$

We can choose the value of Ω_c to be 1.018. Solutions to the following equation gives us the poles of the transfer function:

$$1 + \left(\frac{s_L}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s_L}{j1.018}\right)^{56} = 0$$

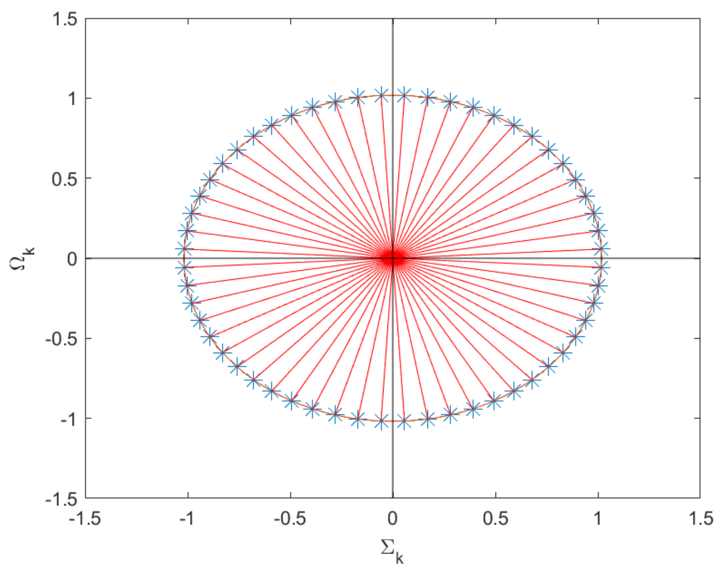

Figure 1: Poles of the Butterworth Transfer Function

In order to get a stable lowpass filter, we must only include poles in the open-LHP.

$$p_1 = -1.0164 - 0.0570797j$$
$$p_2 = -1.0164 + 0.0570797j$$
$$p_3 = -1.00362 - 0.170521j$$
$$p_4 = -1.00362 + 0.170521j$$
$$p_5 = -0.978214 - 0.281819j$$

$$p_6 = -0.978214 + 0.281819j$$
$$p_7 = -0.940509 - 0.389572j$$
$$p_8 = -0.940509 + 0.389572j$$
$$p_9 = -0.890977 + 0.492426j$$
$$p_{10} = -0.890977 - 0.492426j$$
$$p_{11} = -0.830241 - 0.589087j$$
$$p_{12} = -0.830241 + 0.589087j$$
$$p_{13} = -0.759064 - 0.678341j$$
$$p_{14} = -0.759064 + 0.678341j$$
$$p_{15} = -0.678341 - 0.759064j$$
$$p_{16} = -0.678341 + 0.759064j$$
$$p_{17} = -0.589087 - 0.830241j$$
$$p_{18} = -0.589087 + 0.830241j$$
$$p_{19} = -0.492426 - 0.890977j$$
$$p_{20} = -0.492426 + 0.890977j$$
$$p_{21} = -0.389572 - 0.940509j$$
$$p_{22} = -0.389572 + 0.940509j$$
$$p_{23} = -0.281819 - 0.978214j$$
$$p_{24} = -0.281819 + 0.978214j$$
$$p_{25} = -0.170521 - 1.00362j$$
$$p_{26} = -0.170521 + 1.00362j$$
$$p_{27} = -0.0570797 + 1.0164j$$
$$p_{28} = -0.0570797 - 1.0164j$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = \frac{\Omega_c^N}{\prod_{i=1}^{28}(s_L - p_i)}$$

The table given below contains the coefficients for the denominator of $H_{analog,LPF}(s_L)$.

Degree	s_{28}	s_{27}	s_{26}	s_{25}	s_{24}	s_{23}	s_{22}
Coefficient	1	18.1557	164.815	995.348	4489.23	16094.2	47667.5

Degree	s_{21}	s_{20}	s_{19}	s_{18}	s_{17}	s_{16}	s_{15}
Coefficient	119687	259441	491930	823820	1227130	1634050	1951758

Degree	s_{14}	s_{13}	s_{12}	s_{11}	s_{10}	s_9	s_8
Coefficient	2095170	2022654	1754916	1365769	950200	588005	321375

Degree	s_7	s_6	s_5	s_4	s_3	s_2	s_1	s_0
Coefficient	153645	63414.3	22188.6	6413.97	1473.75	252.897	28.8706	1.64792

Table 1: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

2.6 Butterworth Analog Bandpass Transfer Function

The transformation between lowpass and bandpass is given by:

$$\begin{aligned} s_L &= \frac{s^2 + \Omega_0^2}{Bs} \\ &= \frac{s^2 + 0.4775}{2.0334s} \end{aligned}$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BPF}(s)$. Suppose $H_{analog,BPF}(s)$ is represented as $N(s)/D(s)$, we have $N(s) = s^{28}$ and $D(s)$ has the following coefficients:

Degree	s_{56}	s_{55}	s_{54}	s_{53}	s_{52}	s_{51}	s_{50}
Coefficient	1.42e-09	5.24e-08	9.87e-07	1.25e-05	0.00012	0.00094	0.006113

Degree	s_{49}	s_{48}	s_{47}	s_{46}	s_{45}	s_{44}	s_{43}
Coefficient	0.03402	0.16527	0.70981	2.72202	9.39029	29.3080	83.1204

Degree	s_{42}	s_{41}	s_{40}	s_{39}	s_{38}	s_{37}	s_{36}
Coefficient	214.914	507.803	1098.28	2176.49	3953.82	6583.80	10044.5

Degree	s_{35}	s_{34}	s_{33}	s_{32}	s_{31}	s_{30}	s_{29}
Coefficient	14028.7	17916.1	20893.9	22217.7	21509.6	18933.1	15134.9

Degree	s_{28}	s_{27}	s_{26}	s_{25}	s_{24}	s_{23}	s_{22}
Coefficient	10979.7	7226.67	4316.53	2341.54	1154.84	518.561	212.315

Degree	s_{21}	s_{20}	s_{19}	s_{18}	s_{17}	s_{16}	s_{15}
Coefficient	79.3800	27.1381	8.49340	2.43544	0.64013	0.15423	0.03405

Degree	s_{14}	s_{13}	s_{12}	s_{11}	s_{10}	s_9	s_8
Coefficient	0.00688	0.00127	0.00021	3.27e-05	4.53e-06	5.64e-07	6.27e-08

Degree	s_7	s_6	s_5	s_4	s_3	s_2	s_1
Coefficient	6.16e-09	5.28e-10	3.88e-11	2.39e-12	1.18e-13	4.44e-15	1.12e-16

Degree	s_0
Coefficient	1.45e-18

Table 2: Coefficients for the denominator ($D(s)$) of $H_{analog,BPF}(s)$

2.7 Butterworth Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BPF}(s)$, we get $H_{discrete,BPF}(z)$. Suppose $H_{discrete,BPF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	z^0	z^{-2}	z^{-4}	z^{-6}	z^{-8}	z^{-10}	z^{-12}	z^{-14}
Coefficient	1	-28	378	-3276	20475	-98280	376740	-1184040

Degree	z^{-16}	z^{-18}	z^{-20}	z^{-22}	z^{-24}	z^{-26}	z^{-28}
Coefficient	3108105	-6906900	13123110	-21474180	30421755	-37442160	40116600

Degree	z^{-30}	z^{-32}	z^{-34}	z^{-36}	z^{-38}	z^{-40}	z^{-42}
Coefficient	-37442160	30421755	-21474180	13123110	-6906900	3108105	-1184040

Degree	z^{-44}	z^{-46}	z^{-48}	z^{-50}	z^{-52}	z^{-54}	z^{-56}
Coefficient	376740	-98280	20475	-3276	378	-28	1

Table 3: Coefficients for the denominator $(N(z))$ of $H_{discrete,BPF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}
Coefficient	182207	-1424275	4372394	-5912085	1825030	400202	11697284

Degree	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}	z^{-12}	z^{-13}
Coefficient	-22430163	4386676	14608762	12067660	-40040974	4269092	31941509

Degree	z^{-14}	z^{-15}	z^{-16}	z^{-17}	z^{-18}	z^{-19}	z^{-20}
Coefficient	7578927	-43163451	18035	33093507	5676867	-29541681	-3669619

Degree	z^{-21}	z^{-22}	z^{-23}	z^{-24}	z^{-25}	z^{-26}	z^{-27}
Coefficient	19501109	4634754	-12730726	-3346987	6816399	2442148	-3349430

Degree	z^{-28}	z^{-29}	z^{-30}	z^{-31}	z^{-32}	z^{-33}	z^{-34}
Coefficient	-1412880	1392809	732270	-507923	-321754	154836	123016

Degree	z^{-35}	z^{-36}	z^{-37}	z^{-38}	z^{-39}	z^{-40}	z^{-41}
Coefficient	-38936.6	-40211.4	7421.72	11230.6	-848.321	-2642.17	-49.0077

Degree	z^{-42}	z^{-43}	z^{-44}	z^{-45}	z^{-46}	z^{-47}	z^{-48}
Coefficient	516.007	53.2993	-81.5897	-15.1022	10.0542	2.74369	-0.90113

Degree	z^{-49}	z^{-50}	z^{-51}	z^{-52}	z^{-53}	z^{-54}	z^{-55}
Coefficient	-0.34903	0.05003	0.03049	-0.00071	-0.00166	-0.00010	4.29e-05

Degree	z^{-56}
Coefficient	5.48e-06

Table 4: Coefficients for the denominator $(D(z))$ of $H_{discrete,BPF}(z)$

2.8 Chebyshev Analog Lowpass Transfer Function

Based on the tolerance in the passband (δ_p) and the stopband (δ_s), we define the following quantities:

$$D_1 = \frac{1}{(1-\delta_p)^2} - 1 = \frac{1}{0.922^2} - 1 = 0.1765$$
$$D_2 = \frac{1}{\delta_s^2} - 1 = \frac{1}{0.15^2} - 1 = 43.4444$$

Using these newly defined quantities, the minimum order for the Chebyshev filter is given as:

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{D_2/D_1})}{cosh^{-1}(\Omega_{L_s}/\Omega_{L_p})} \rceil = \lceil 8.1803 \rceil = 9$$

Solutions to the following equation gives us the poles of the transfer function:

$$1 + D_1 cosh^2\left(N cosh^{-1}\left(\frac{s_L}{j\Omega_{L_p}}\right)\right) = 1 + 0.1765 cosh^2\left(9 cosh^{-1}\left(\frac{s_L}{j}\right)\right) = 0$$

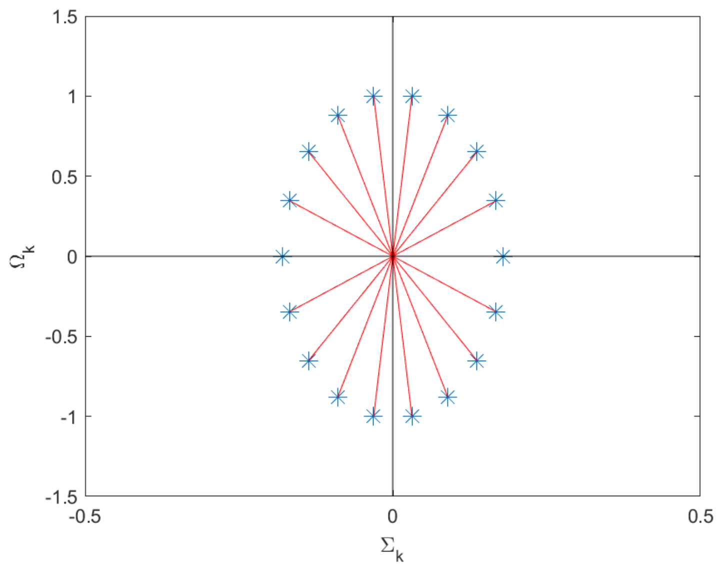

Figure 2: Poles of the Chebyshev Transfer Function

In order to get a stable lowpass filter, we must only include poles in the open-LHP.

$$p_1 = -0.03107 - 1.00045j$$
$$p_2 = -0.03107 + 1.00045j$$
$$p_3 = -0.08946 - 0.87978j$$
$$p_4 = -0.08946 + 0.87978j$$
$$p_5 = -0.13706 - 0.65300j$$
$$p_6 = -0.13706 + 0.65300j$$
$$p_7 = -0.16813 - 0.34745j$$
$$p_8 = -0.16813 + 0.34745j$$
$$p_9 = -0.17892$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = \frac{\prod_{i=1}^{9} p_i}{\prod_{i=1}^{9}(s_L - p_i)} = \frac{-0.0093}{\prod_{i=1}^{9}(s_L - p_i)}$$

The table given below contains the coefficients for the denominator of $H_{analog,LPF}(s_L)$.

Degree	s_9	s_8	s_7	s_6	s_5
Coefficient	1	1.03037	2.78083	2.14978	2.56730

Degree	s_3	s_3	s_2	s_1	s_0
Coefficient	1.39820	0.87781	0.28968	0.08138	0.009298

Table 5: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

2.9 Chebyshev Analog Bandpass Transfer Function

The transformation between lowpass and bandpass is given by:

$$\begin{aligned} s_L &= \frac{s^2 + \Omega_0^2}{Bs} \\ &= \frac{s^2 + 0.4775}{2.0334s} \end{aligned}$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BPF}(s)$. Suppose $H_{analog,BPF}(s)$ is represented as $N(s)/D(s)$, we have $N(s) = s^9$ and $D(s)$ has the following coefficients:

Degree	s_{18}	s_{17}	s_{16}	s_{15}	s_{14}	s_{13}
Coefficient	-0.18091	-0.37904	-2.85751	-4.71780	-16.3774	-20.5808

Degree	s_{12}	s_{11}	s_{10}	s_9	s_8	s_7
Coefficient	-41.7941	-37.8187	-47.5911	-28.7158	-22.7205	-8.61965

Degree	s_6	s_5	s_4	s_3	s_2	s_1	s_0
Coefficient	-4.54767	-1.06912	-0.40616	-0.05586	-0.01615	-0.00102	-0.00023

Table 6: Coefficients for the denominator $(D(s))$ of $H_{analog,BPF}(s)$

2.10 Chebyshev Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BPF}(s)$, we get $H_{discrete,BPF}(z)$. Suppose $H_{discrete,BPF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	z^0	z^{-2}	z^{-4}	z^{-6}	z^{-8}	z^{-10}	z^{-12}	z^{-14}	z^{-16}	z^{-18}
Coefficient	-1	9	-36	84	-126	126	-84	36	-9	1

Table 7: Coefficients for the denominator $(N(z))$ of $H_{discrete,BPF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}
Coefficient	238.450	-810.845	1435.53	-2060.80	3024.83	-4071.51	4775.51

Degree	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}	z^{-12}	z^{-13}
Coefficient	-5147.73	5309.76	-5081.28	4488.49	-3681.63	2821.27	-1975.18

Degree	z^{-14}	z^{-15}	z^{-16}	z^{-17}	z^{-18}
Coefficient	1256.41	-711.104	357.456	-141.082	34.5338

Table 8: Coefficients for the denominator ($D(z)$) of $H_{discrete,BPF}(z)$

2.11 Elliptical Analog Lowpass Transfer Function

2.11.1 Jacobian Elliptical Integrals

The elliptical function $\omega = sn(z,k)$ can be defined using the elliptical integral:

$$z = \int_0^{\phi} \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}$$

Using a change of variables, we get

$$z = \int_0^{\omega} \frac{dt}{\sqrt{(1-k^2t^2)(1-t^2)}}$$

where $\omega = sin(\phi(z,k))$ and k is called the elliptic modulus with $0 \leq k \leq 1$.
The three elliptical functions cn, dn, and cd are defined as follows:

$$\omega = cn(z,k) = cos\phi(z,k)$$

$$\omega = dn(z,k) = \frac{d}{dz}\phi(z,k)$$

$$\omega = cd(z,k) = \frac{cn\phi(z,k)}{dn\phi(z,k)}$$

The complete elliptical integral is defined as the value of z at $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}$$

at $\phi = \pi/2$, the elliptical functions are defined as

$$sn(K,k) = 1 \quad \& \quad cd(K,k) = 0$$

The complementary elliptical modulus $k' = \sqrt{1-k^2}$ can also be used to define the complete elliptical integral

$$K(k') = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k'^2 sin^2\theta}}$$

2.11.2 Elliptical Filter Parameters

Based on the tolerance in the passband (δ_p) and the stopband (δ_s), we define the following quantities:

$$D_1 = \sqrt{\frac{1}{(1-\delta_p)^2} - 1} = \sqrt{\frac{1}{(0.922)^2} - 1} = 0.42$$

$$D_2 = \sqrt{\frac{1}{(\delta_s)^2} - 1} = \sqrt{\frac{1}{(0.15)^2} - 1} = 6.591$$

$$k_1 = \frac{D_1}{D_2} = \frac{0.42}{6.591} = 0.0637$$

$$k_1' = \sqrt{1-k_1^2} = 0.998$$

$$k = \frac{\Omega_{L_p}}{\Omega_{L_s}} = \frac{1}{1.09} = 0.9174$$

$$k' = \sqrt{1-k^2} = 0.398$$

Using these newly defined quantities, the minimum order for the Elliptical filter is given as:

$$N_{min} = \lceil \frac{K(k) \times K(k_1')}{K(k') \times K(k_1)} \rceil$$

where

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}$$

The required elliptical integral values can be calculated using MATLAB:

$$K(k) = 2.3641 \qquad K(k_1') = 4.1429 \qquad K(k') = 1.6392 \qquad K(k_1) = 1.5724$$

$$N_{min} = \lceil 3.8 \rceil = 4$$

2.11.3 Poles and Zeroes

We define L and r as follows:

$$L = \lfloor \frac{N}{2} \rfloor \quad \& \quad r = mod(N, 2)$$

$$u_i = \frac{2i-1}{N} \qquad i = 1, 2....L$$

$$\zeta_i = cd(u_i, k)$$

The zeroes of the transfer function $H_{analog,LPF}(s_L)$ are given as follows:

$$z_i = j\Omega_i = \frac{j}{k \cdot \zeta_i} \qquad i = 1, 2....L$$

We define ν_0 as follows:

$$\nu_0 = -\frac{j}{N} sn^{-1}(\frac{j}{D_1}, k_1)$$

The poles of the transfer function $H_{analog,LPF}(s_L)$ are given as follows:

$$p_i = j \cdot cd((u_i - j\nu_0), k) \qquad i = 1, 2....L$$

Since N is odd, there is an additional pole given by

$$p_0 = j \cdot cd((1 - j\nu_0), k) = j \cdot sn(j\nu_0, k)$$

The poles and zeroes are given as follows:

$$z_1 = 1.09969j$$
$$z_2 = -1.09969j$$
$$z_3 = 1.93712j$$
$$z_4 = -1.93712j$$

$$p_1 = -0.0481 + 1.01142j$$
$$p_2 = -0.0481 - 1.01142j$$
$$p_3 = -0.44991 + 0.71947j$$
$$p_4 = -0.44991 - 0.71947j$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = 0.922 \frac{\left(\prod_{i=1}^4 p_i\right)\left(\prod_{i=1}^4 (s_L - z_i)\right)}{\left(\prod_{i=1}^4 z_i\right)\left(\prod_{i=1}^4 (s_L - p_i)\right)}$$

The tables given below contains the coefficients for the numerator and denominator of $H_{analog,LPF}(s_L)$.

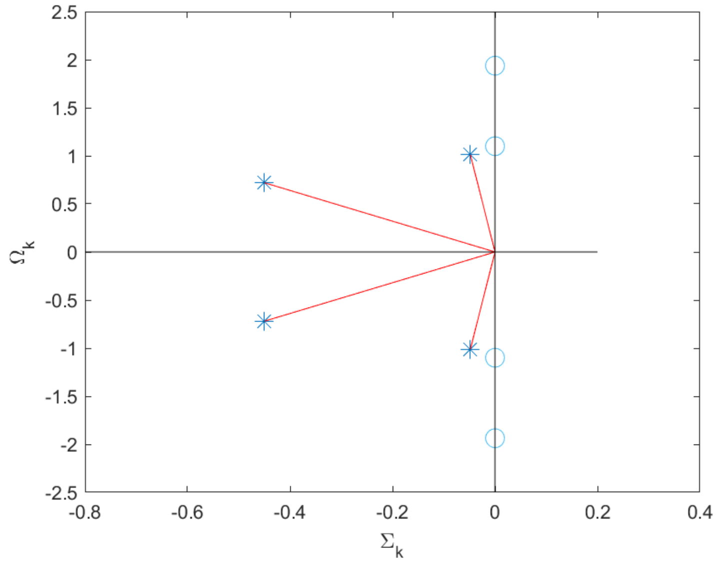

Figure 3: Poles and Zeroes of the Elliptical Transfer Function

Degree	s_4	s_2	s_0
Coefficient	0.15	0.74426	0.68068

Table 9: Coefficients for the numerator of $H_{analog,LPF}(s_L)$

2.12 Elliptical Analog Bandpass Transfer Function

The transformation between lowpass and bandpass is given by:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$
$$= \frac{s^2 + 0.4775}{2.0334s}$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BPF}(s)$. Suppose $H_{analog,BPF}(s)$ is represented as $N(s)/D(s)$, we have $N(s)$ and $D(s)$ have the following coefficients:

2.13 Elliptical Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BPF}(s)$, we get $H_{discrete,BPF}(z)$. Suppose $H_{discrete,BPF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	1	0.99602	1.8319	0.99184	0.73826

Table 10: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

Degree	s_8	s_6	s_4	s_2	s_0
Coefficient	0.00787	0.1764	0.77513	0.0402	0.00041

Table 11: Coefficients for the numerator $(N(s))$ of $H_{analog,BPF}(s)$

Degree	s_8	s_7	s_6	s_5
Coefficient	0.05244	0.1062	0.49734	0.58942

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	1.11288	0.28139	0.11335	0.01156	0.00272

Table 12: Coefficients for the denominator $(D(s))$ of $H_{analog,BPF}(s)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	1	-0.60442	-2.00243	0.12718

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	3.06394	0.12718	-2.00243	-0.60442	1

Table 13: Coefficients for the numerator $(N(z))$ of $H_{discrete,BPF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	2.76731	-3.11756	-0.55727	-0.72486

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	4.43159	-1.77104	-0.37123	-0.74971	0.79016

Table 14: Coefficients for the denominator $(D(z))$ of $H_{discrete,BPF}(z)$

2.14 FIR Filter Parameters

Based on the required filter specifications, we define the following quantities:

$$A = -20 \log_{10} \delta = -20 \log_{10} 0.15 = 16.4782$$

Since $A < 21$, we take $\alpha = \beta = 0$, and therefore the Kaiser window will be rectangular in shape. The minimum width of the Kaiser window can be calculated using:

$$M \geq 1 + \frac{A - 8}{2.285 \Delta\omega_t} = 71.8627$$

where $\Delta\omega_t$ is the transition bandwidth, i.e., $5\pi/300$. We take the next odd integer value, i.e., 73. However, according to simulations, $M = 91$ is the minimum window width which properly meets specifications.

2.15 FIR Discrete Time Filter

The coefficients for the obtained FIR filter are given as follows:

```
Columns 1 through 18

  0.0000    0.0138    0.0044    0.0019    0.0104   -0.0069   -0.0012   -0.0014   -0.0164   -0.0017   -0.0096   -0.0125    0.0059   -0.0074    0.0042    0.0150   -0.0008    0.0190

Columns 19 through 36

  0.0118   -0.0009    0.0175   -0.0078   -0.0078    0.0019   -0.0287   -0.0080   -0.0107   -0.0283    0.0105   -0.0081    0.0000    0.0373    0.0005    0.0347    0.0405   -0.0082

Columns 37 through 54

  0.0437   -0.0083   -0.0432    0.0149   -0.1093   -0.0713   -0.0291   -0.2715    0.1091    0.6167    0.1091   -0.2715   -0.0291   -0.0713   -0.1093    0.0149   -0.0432   -0.0083

Columns 55 through 72

  0.0437   -0.0082    0.0405    0.0347    0.0005    0.0373    0.0000   -0.0081    0.0105   -0.0283   -0.0107   -0.0080   -0.0287    0.0019   -0.0078   -0.0078    0.0175   -0.0009

Columns 73 through 90

  0.0118    0.0190   -0.0008    0.0150    0.0042   -0.0074    0.0059   -0.0125   -0.0096   -0.0017   -0.0164   -0.0014   -0.0012   -0.0069    0.0104    0.0019    0.0044    0.0138

Column 91

  0.0000
```

Figure 4: Coefficients of $H_{discrete,BPF}(z)$

3 Bandstop Filter Details

3.1 Un-normalized Discrete-time Filter Specifications

Given below are the filter specifications for the required bandpass filter:

- Stopband : 75 - 185 kHz
- Passband : 0 - 70 kHz and 190 - 300 kHz
- Transition band : 5 kHz on either sides of the stopband
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

3.2 Normalized Digital Filter Specifications

Sampling Rate = 600 kHz corresponds to 2π on the normalized frequency axis.

$$f_s \rightarrow 2\pi$$

$$\omega = 2\pi \times f/f_s$$

Therefore the normalized discrete filter specifications are as follows:

- Stopband : $75\pi/300$ - $185\pi/300$
- Passband : 0 - $70\pi/300$ and $190\pi/300$ - π
- Transition band : $5\pi/300$ on either sides of the stopband
- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.
- Passband nature : Monotonic
- Stopband nature : Monotonic

3.3 Analog Filter Specifications

The bilinear transformation is given as:

$$\Omega = tan(\omega/2)$$

Therefore the corresponding analog filter specifications are as follows:

- Stopband : 0.4142 (Ω_{s_1}) - 1.455 (Ω_{s_2})
- Passband : 0 - 0.3839 (Ω_{p_1}) and 1.5399 (Ω_{p_2}) - ∞
- Transition band : 0.3839 - 0.4142 and 1.455 - 1.5399

- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.

- Passband nature : Monotonic

- Stopband nature : Monotonic

3.4 Frequency-transformed Lowpass Analog Filter

The bandpass transformation is given as follows:

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p_1}\Omega_{p_2}} = 0.7689$$
$$B = \Omega_{p_2} - \Omega_{p_1} = 1.156$$

The lowpass transformations for various key points are given below:

Ω	Ω_L
0^+	-0^+
0.3839 (Ω_{p_1})	1 ($\Omega_{L_{p_1}}$)
0.4142 (Ω_{s_1})	1.1411 ($\Omega_{L_{s_1}}$)
0.7689^- (Ω_0^-)	∞
0.7689^+ (Ω_0^+)	$-\infty$
1.455 (Ω_{s_2})	-1.1024 ($\Omega_{L_{s_2}}$)
1.5399 (Ω_{p_2})	-1 ($\Omega_{L_{p_2}}$)

Therefore the corresponding lowpass analog filter specifications are as follows:

- Passband Edge : 1 (Ω_{L_p})

- Stopband Edge : $\min(|\Omega_{L_{s_1}}|, |\Omega_{L_{s_2}}|) = 1.1024$ (Ω_{L_s})

- Tolerance : 0.15 in magnitude for both oscillatory and monotonic stopband.
  0.078 for oscillatory passband and 0.15 for a monotonic passband. This difference is because in case of an oscillatory passband, after cascading the effective tolerance will be $(1-\delta_p)^2 > 0.85$. Therefore, $\delta_p < 0.078$.

- Passband nature : Monotonic

- Stopband nature : Monotonic

3.5 Butterworth Analog Lowpass Transfer Function

Based on the tolerance in the passband and the stopband (both equal to δ), we define two new quantities:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.4444$$

Using these newly defined quantities, the minimum order for the Butterworth filter is given as:

$$N_{min} = \lceil \frac{log(D_2/D_1)}{2log(\Omega_{L_s}/\Omega_{L_p})} \rceil = \lceil 24.2501 \rceil = 25$$

The cutoff frequency (Ω_c) of the analog lowpass analog filter has the following constraint:

$$\frac{\Omega_{L_p}}{D_1^{1/2N}} \leq \Omega_c \leq \frac{\Omega_{L_s}}{D_2^{1/2N}}$$

$$1.0193 \leq \Omega_c \leq 1.0269$$

We can choose the value of Ω_c to be 1.02. Solutions to the following equation gives us the poles of the transfer function:

$$1 + \left(\frac{s_L}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s_L}{j1.02}\right)^{50} = 0$$

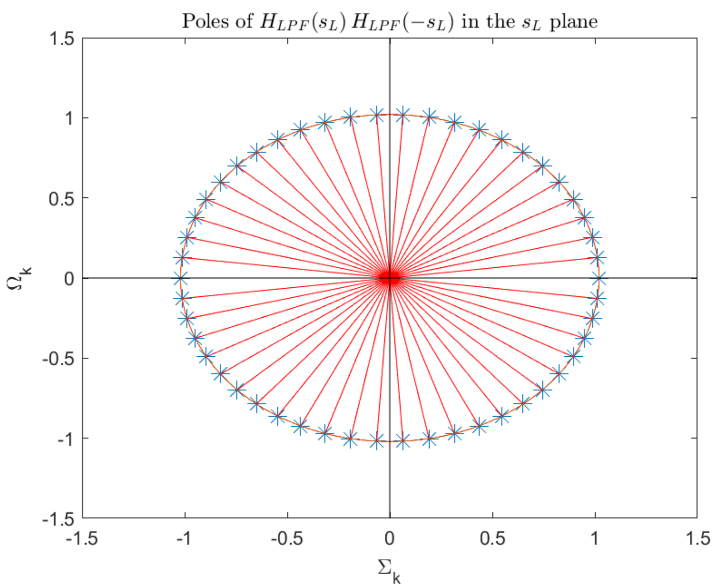

Figure 5: Poles of the Butterworth Transfer Function

In order to get a stable lowpass filter, we must only include poles in the open-LHP.

$$p_1 = -1.02$$
$$p_2 = -1.01196 - 0.12784j$$
$$p_3 = -1.01196 + 0.12784j$$
$$p_4 = -0.987955 - 0.253664j$$
$$p_5 = -0.987955 + 0.253664j$$
$$p_6 = -0.948372 - 0.375487j$$
$$p_7 = -0.948372 + 0.375487j$$
$$p_8 = -0.893833 - 0.491389j$$
$$p_9 = -0.893833 + 0.491389j$$
$$p_{10} = -0.825197 - 0.599541j$$

$$p_{11} = -0.825197 + 0.599541j$$
$$p_{12} = -0.743548 - 0.698238j$$
$$p_{13} = -0.743548 + 0.698238j$$
$$p_{14} = -0.650172 - 0.785924j$$
$$p_{15} = -0.650172 + 0.785924j$$
$$p_{16} = -0.546543 - 0.861214j$$
$$p_{17} = -0.546543 + 0.861214j$$
$$p_{18} = -0.434295 - 0.922924j$$
$$p_{19} = -0.434295 + 0.922924j$$
$$p_{20} = -0.315197 - 0.970078j$$
$$p_{21} = -0.315197 + 0.970078j$$
$$p_{22} = -0.191129 - 1.00193j$$
$$p_{23} = -0.191129 + 1.00193j$$
$$p_{24} = -0.0640463 - 1.01799j$$
$$p_{25} = -0.0640463 + 1.01799j$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = \frac{\Omega_c^N}{\prod_{i=1}^{25}(s_L - p_i)}$$

The table given below contains the coefficients for the denominator of $H_{analog,LPF}(s_L)$.

Degree	s_{25}	s_{24}	s_{23}	s_{22}	s_{21}	s_{20}	s_{19}
Coefficient	1	16.2444	131.941	712.554	2870.77	9178.10	24186.0

Degree	s_{18}	s_{17}	s_{16}	s_{15}	s_{14}	s_{13}	s_{12}
Coefficient	53871.5	103204	172159	252246	326553	374913	382411

Degree	s_{11}	s_{10}	s_9	s_8	s_7	s_6	s_5
Coefficient	346540	278500	197757	123339	66982.4	31287.2	12352.5

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	4019.77	1038.05	199.979	25.6159	1.64060

Table 15: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

3.6 Butterworth Analog Bandstop Transfer Function

The transformation between lowpass and bandstop is given by:

$$s_L = \frac{Bs}{s^2 + \Omega_0^2} = \frac{1.156s}{s^2 + 0.7689}$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BSF}(s)$. Suppose $H_{analog,BSF}(s)$ is represented as $N(s)/D(s)$, we have $N(s)$ and $D(s)$ have the following coefficients:

Degree	s_{50}	s_{48}	s_{46}	s_{44}	s_{42}	s_{40}	s_{38}
Coefficient	9.06925	0.00013	0.00095	0.00430	0.01400	0.03477	0.06850

Degree	s_{36}	s_{34}	s_{32}	s_{30}	s_{28}	s_{26}	s_{24}
Coefficient	0.10991	0.14618	0.16322	0.15436	0.12442	0.08580	0.05072

Degree	s_{22}	s_{20}	s_{18}	s_{16}	s_{14}	s_{12}	s_{10}
Coefficient	0.02569	0.01113	0.00411	0.00128	0.00033	7.36e-05	1.30e-05

Degree	s_8	s_6	s_4	s_2	s_0
Coefficient	1.83e-06	1.97e-07	1.52e-08	7.50e-10	1.77e-11

Table 16: Coefficients for the numerator ($N(s)$) of $H_{analog,BSF}(s)$

3.7 Butterworth Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BSF}(s)$, we get $H_{discrete,BSF}(z)$. Suppose $H_{discrete,BSF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	s_{50}	s_{49}	s_{48}	s_{47}	s_{46}	s_{45}	s_{44}
Coefficient	9.06e-06	0.00016	0.00161	0.01118	0.06071	0.27202	1.04022

Degree	s_{43}	s_{42}	s_{41}	s_{40}	s_{39}	s_{38}	s_{37}
Coefficient	3.47189	10.2758	27.2885	65.6029	143.767	288.785	534.032

Degree	s_{36}	s_{35}	s_{34}	s_{33}	s_{32}	s_{31}	s_{30}
Coefficient	912.365	1444.17	2122.87	2903.31	3699.87	4398.73	4883.52

Degree	s_{29}	s_{28}	s_{27}	s_{26}	s_{25}	s_{24}	s_{23}
Coefficient	5066.74	4915.43	4460.82	3788.00	3010.38	2239.08	1558.60

Degree	s_{22}	s_{21}	s_{20}	s_{19}	s_{18}	s_{17}	s_{16}
Coefficient	1015.17	618.541	352.398	187.623	93.2840	43.2687	18.7009

Degree	s_{15}	s_{14}	s_{13}	s_{12}	s_{11}	s_{10}	s_9
Coefficient	7.52004	2.80821	0.97160	0.31056	0.09139	0.02465	0.00606

Degree	s_8	s_7	s_6	s_5	s_4	s_3	s_2
Coefficient	0.00134	0.00026	4.77e-05	7.37e-06	9.73e-07	1.05e-07	9.02e-09

Degree	s_1	s_0
Coefficient	5.41e-10	1.77e-11

Table 17: Coefficients for the denominator $(D(s))$ of $H_{analog,BSF}(s)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}
Coefficient	1	-12.8496	104.254	-620.698	3005.72	-12323.1	44156.9

Degree	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}	z^{-12}	z^{-13}
Coefficient	-140823	405721	-1066723	2581279	-5785163	12073338	-23561016

Degree	z^{-14}	z^{-15}	z^{-16}	z^{-17}	z^{-18}	z^{-19}
Coefficient	43150517	-74381598	120983099	-186059322	271026690	-374478160

Degree	z^{-20}	z^{-21}	z^{-22}	z^{-23}	z^{-24}	z^{-25}
Coefficient	491385660	-612935697	727351418	-821594803	883767338	-905492148

Degree	z^{-26}	z^{-27}	z^{-28}	z^{-29}	z^{-30}	z^{-31}
Coefficient	883767338	-821594803	727351418	-612935697	491385660	-374478160

Degree	z^{-32}	z^{-33}	z^{-34}	z^{-35}	z^{-36}	z^{-37}
Coefficient	271026690	-186059322	120983099	-74381598	43150517	-23561016

Degree	z^{-38}	z^{-39}	z^{-40}	z^{-41}	z^{-42}	z^{-43}	z^{-44}
Coefficient	12073338	-5785163	2581279	-1066723	405721	-140823	44156.9

Degree	z^{-45}	z^{-46}	z^{-47}	z^{-48}	z^{-49}	z^{-50}
Coefficient	-12323.1	3005.72	-620.698	104.254	-12.8496	1

Table 18: Coefficients for the numerator $(N(z))$ of $H_{discrete,BSF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}	z^{-6}
Coefficient	48819.2	-380094	1688980	-5472779	14384359	-32328276	64159832

Degree	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}	z^{-12}
Coefficient	-114750228	187637343	-283439139	398669067	-525292603	651505460

Degree	z^{-13}	z^{-14}	z^{-15}	z^{-16}	z^{-17}	z^{-18}
Coefficient	-763524799	848149351	-895284994	899899328	-862787039	790118905

Degree	z^{-19}	z^{-20}	z^{-21}	z^{-22}	z^{-23}	z^{-24}
Coefficient	-691896295	579873208	-465435054	357958520	-263869984	186466566

Degree	z^{-25}	z^{-26}	z^{-27}	z^{-28}	z^{-29}	z^{-30}
Coefficient	-126318233	82020803	-51032367	30411634	-17347734	9465060

Degree	z^{-31}	z^{-32}	z^{-33}	z^{-34}	z^{-35}	z^{-36}	z^{-37}
Coefficient	-4934781	2455742	-1164858	525828	-225459	91618.2	-35191.4

Degree	z^{-38}	z^{-39}	z^{-40}	z^{-41}	z^{-42}	z^{-43}	z^{-44}
Coefficient	12737.8	-4328.44	1374.78	-405.909	110.677	-27.6316	6.25019

Degree	z^{-45}	z^{-46}	z^{-47}	z^{-48}	z^{-49}	z^{-50}
Coefficient	-1.26198	0.22316	-0.03349	0.00408	-0.00036	2.04e-05

Table 19: Coefficients for the denominator ($D(z)$) of $H_{discrete,BSF}(z)$

3.8 Chebyshev Analog Lowpass Transfer Function

Based on the tolerance in the passband (δ_p) and the stopband (δ_s), we define two new quantities:

$$D_1 = \frac{1}{(1-\delta_p)^2} - 1 = \frac{1}{0.922^2} - 1 = 0.1765$$

$$D_2 = \frac{1}{\delta_s^2} - 1 = \frac{1}{0.15^2} - 1 = 43.4444$$

Using these newly defined quantities, the minimum order for the Chebyshev filter is given as:

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{D_2/D_1})}{cosh^{-1}(\Omega_{L_s}/\Omega_{L_p})} \rceil = \lceil 7.6767 \rceil = 8$$

Solutions to the following equation gives us the poles of the transfer function:

$$1 + D_1 cosh^2\left(N cosh^{-1}\left(\frac{s_L}{j\Omega_{L_p}}\right)\right) = 1 + 0.1765 cosh^2\left(8 cosh^{-1}\left(\frac{s_L}{j}\right)\right) = 0$$

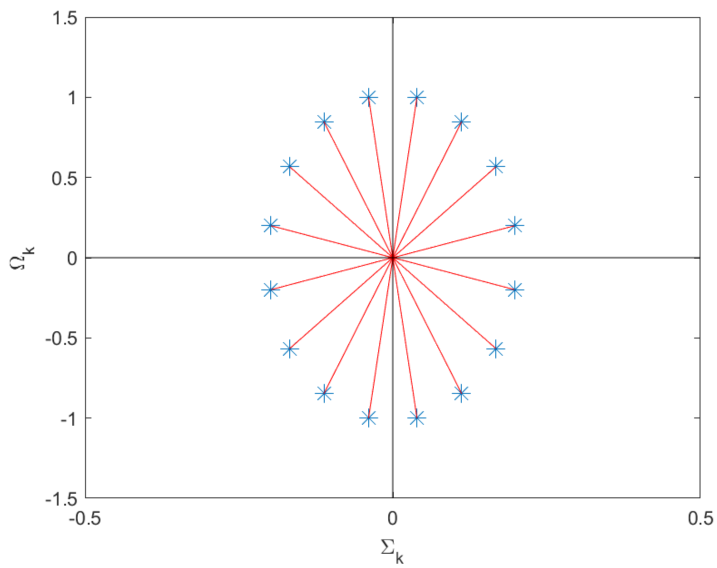

Figure 6: Poles of the Chebyshev Transfer Function

In order to get a stable lowpass filter, we must only include poles in the open-LHP.

$$p_1 = -0.03932 - 1.00051j$$
$$p_2 = -0.03932 + 1.00051j$$
$$p_3 = -0.11199 - 0.84819j$$
$$p_4 = -0.11199 + 0.84819j$$
$$p_5 = -0.16760 - 0.56674j$$
$$p_6 = -0.16760 + 0.56674j$$
$$p_7 = -0.19770 - 0.19901j$$
$$p_8 = -0.19770 + 0.19901j$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = \frac{\prod_{i=1}^{8} p_i}{\sqrt{(1+D_1)}\prod_{i=1}^{8}(s_L - p_i)} = \frac{0.0186}{\prod_{i=1}^{8}(s_L - p_i)}$$

The table given below contains the coefficients for the denominator of $H_{analog,LPF}(s_L)$.

Degree	s_8	s_7	s_6	s_5	s_4
Coefficient	1	1.03321	2.53376	1.89632	1.99933

Degree	s_3	s_2	s_1	s_0
Coefficient	0.99079	0.50590	0.12846	0.02017

Table 20: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

3.9 Chebyshev Analog Bandstop Transfer Function

The transformation between lowpass and bandstop is given by:

$$
\begin{aligned}
s_L &= \frac{Bs}{s^2 + \Omega_0^2} \\
&= \frac{1.156s}{s^2 + 0.7689}
\end{aligned}
$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BSF}(s)$. Suppose $H_{analog,BSF}(s)$ is represented as $N(s)/D(s)$, we have $N(s)$ and $D(s)$ have the following coefficients:

Degree	s_{16}	s_{14}	s_{12}	s_{10}	s_8
Coefficient	0.02435	0.11513	0.23818	0.28157	0.20805

Degree	s_6	s_4	s_2	s_0
Coefficient	0.09838	0.02908	0.00491	0.00036

Table 21: Coefficients for the numerator ($N(s)$) of $H_{analog,BSF}(s)$

Degree	s_{16}	s_{15}	s_{14}	s_{13}	s_{12}	s_{11}
Coefficient	0.02641	0.19442	1.00995	2.80826	8.07172	12.4740

Degree	s_{10}	s_9	s_8	s_7	s_6	s_5
Coefficient	23.9124	21.2268	27.2145	12.5471	8.35493	2.57623

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	0.98539	0.20265	0.04308	0.00490	0.00039

Table 22: Coefficients for the denominator ($D(s)$) of $H_{analog,BSF}(s)$

3.10 Chebyshev Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BSF}(s)$, we get $H_{discrete,BSF}(z)$. Suppose $H_{discrete,BSF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}
Coefficient	1	-4.11189	15.3971	-36.3872	77.2679	-126.378

Degree	z^{-6}	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}
Coefficient	187.014	-226.058	248.292	-226.058	187.014	-126.378

Degree	z^{-12}	z^{-13}	z^{-14}	z^{-15}	z^{-16}
Coefficient	77.2679	-36.3872	15.3971	-4.11189	1

Table 23: Coefficients for the numerator $(N(z))$ of $H_{discrete,BSF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}	z^{-4}	z^{-5}
Coefficient	121.653	-236.394	162.520	-143.341	359.238	-310.484

Degree	z^{-6}	z^{-7}	z^{-8}	z^{-9}	z^{-10}	z^{-11}
Coefficient	17.0803	-63.9651	235.689	-50.0155	-99.7057	-40.0692

Degree	z^{-12}	z^{-13}	z^{-14}	z^{-15}	z^{-16}
Coefficient	86.1319	17.3447	-21.9911	-25.4840	17.5844

Table 24: Coefficients for the denominator $(D(z))$ of $H_{discrete,BSF}(z)$

3.11 Elliptical Analog Lowpass Transfer Function

3.11.1 Jacobian Elliptical Integrals

The elliptical function $\omega = sn(z, k)$ can be defined using the elliptical integral:

$$z = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 sin^2\theta}}$$

Using a change of variables, we get

$$z = \int_0^{\omega} \frac{dt}{\sqrt{(1 - k^2t^2)(1 - t^2)}}$$

where $\omega = sin(\phi(z, k))$ and k is called the elliptic modulus with $0 \leq k \leq 1$.
The three elliptical functions cn, dn, and cd are defined as follows:

$$\omega = cn(z, k) = cos\phi(z, k)$$

$$\omega = dn(z, k) = \frac{d}{dz}\phi(z, k)$$

$$\omega = cd(z, k) = \frac{cn\phi(z, k)}{dn\phi(z, k)}$$

The complete elliptical integral is defined as the value of z at $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 sin^2\theta}}$$

at $\phi = \pi/2$, the elliptical functions are defined as

$$sn(K, k) = 1 \quad \& \quad cd(K, k) = 0$$

The complementary elliptical modulus $k' = \sqrt{1 - k^2}$ can also be used to define the complete elliptical integral

$$K(k') = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k'^2 sin^2\theta}}$$

3.11.2 Elliptical Filter Parameters

Based on the tolerance in the passband (δ_p) and the stopband (δ_s), we define the following quantities:

$$D_1 = \sqrt{\frac{1}{(1 - \delta_p)^2} - 1} = \sqrt{\frac{1}{(0.922)^2} - 1} = 0.42$$

$$D_2 = \sqrt{\frac{1}{(\delta_s)^2} - 1} = \sqrt{\frac{1}{(0.15)^2} - 1} = 6.591$$

$$k_1 = \frac{D_1}{D_2} = \frac{0.42}{6.591} = 0.0637$$

$$k_1' = \sqrt{1 - k_1^2} = 0.998$$

$$k = \frac{\Omega_{L_p}}{\Omega_{L_s}} = \frac{1}{1.1024} = 0.9071$$

$$k' = \sqrt{1-k^2} = 0.4209$$

Using these newly defined quantities, the minimum order for the Elliptical filter is given as:

$$N_{min} = \lceil \frac{K(k) \times K(k_1')}{K(k') \times K(k_1)} \rceil$$

where

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}$$

The required elliptical integral values can be calculated using MATLAB:

$$K(k) = 2.31254 \qquad K(k_1') = 4.14286 \qquad K(k') = 1.64828 \qquad K(k_1) = 1.57239$$

$$N_{min} = \lceil 3.69654 \rceil = 4$$

3.11.3 Poles and Zeroes

We define L and r as follows:

$$L = \lfloor \frac{N}{2} \rfloor \quad \& \quad r = mod(N,2)$$

$$u_i = \frac{2i-1}{N} \qquad i = 1,2....L$$

$$\zeta_i = cd(u_i, k)$$

The zeroes of the transfer function $H_{analog,LPF}(s_L)$ are given as follows:

$$z_i = j\Omega_i = \frac{j}{k \cdot \zeta_i} \qquad i = 1,2....L$$

We define ν_0 as follows:

$$\nu_0 = -\frac{j}{N} sn^{-1}(\frac{j}{D_1}, k_1)$$

The poles of the transfer function $H_{analog,LPF}(s_L)$ are given as follows:

$$p_i = j \cdot cd((u_i - j\nu_0), k) \qquad i = 1,2....L$$

Since N is odd, there is an additional pole given by

$$p_0 = j \cdot cd((1 - j\nu_0), k) = j \cdot sn(j\nu_0, k)$$

The poles and zeroes are given as follows:

$$z_1 = 1.09969j$$
$$z_2 = -1.09969j$$
$$z_3 = 1.93712j$$
$$z_4 = -1.93712j$$

$$p_1 = -0.0481 + 1.01142j$$
$$p_2 = -0.0481 - 1.01142j$$
$$p_3 = -0.44991 + 0.71947j$$
$$p_4 = -0.44991 - 0.71947j$$

The analog lowpass transfer function can be written as follows:

$$H_{analog,LPF}(s_L) = 0.922 \frac{\left(\prod_{i=1}^{4} p_i\right)\left(\prod_{i=1}^{4}(s_L - z_i)\right)}{\left(\prod_{i=1}^{4} z_i\right)\left(\prod_{i=1}^{4}(s_L - p_i)\right)}$$

The tables given below contains the coefficients for the numerator and denominator of $H_{analog,LPF}(s_L)$.

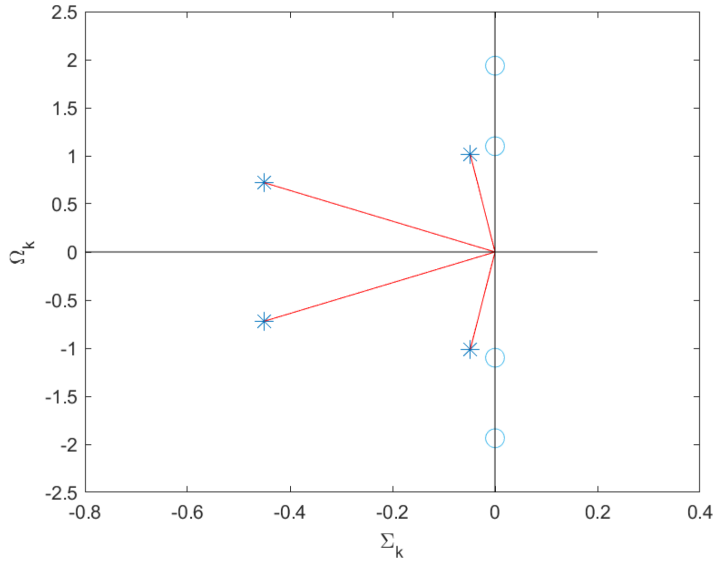

Figure 7: Poles and Zeroes of the Elliptical Transfer Function

Degree	s_4	s_2	s_0
Coefficient	0.15	0.74426	0.68068

Table 25: Coefficients for the numerator of $H_{analog,LPF}(s_L)$

3.12 Elliptical Analog Bandpass Transfer Function

The transformation between lowpass and bandstop is given by:

$$s_L = \frac{Bs}{s^2 + \Omega_0^2}$$
$$= \frac{1.156s}{s^2 + 0.7689}$$

After substituting this value into $H_{analog,LPF}(s_L)$, we get $H_{analog,BSF}(s)$. Suppose $H_{analog,BSF}(s)$ is represented as $N(s)/D(s)$, we have $N(s)$ and $D(s)$ have the following coefficients:

3.13 Elliptical Discrete Time Filter Transfer Function

The transformation of the analog transfer function to the discrete domain is given by the Bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

After substituting this value into $H_{analog,BPF}(s)$, we get $H_{discrete,BPF}(z)$. Suppose $H_{discrete,BPF}(z)$ is represented as $N(z)/D(z)$, the coefficients for $N(z)$ and $D(z)$ are given as follows:

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	1	0.99602	1.8319	0.99184	0.73826

Table 26: Coefficients for the denominator of $H_{analog,LPF}(s_L)$

Degree	s_8	s_6	s_4	s_2	s_0
Coefficient	0.09522	0.36428	0.40159	0.12728	0.01162

Table 27: Coefficients for the numerator $(N(s))$ of $H_{analog,BPF}(s)$

Degree	s_8	s_7	s_6	s_5
Coefficient	0.10328	0.1604	0.58666	0.49968

Degree	s_4	s_3	s_2	s_1	s_0
Coefficient	0.8712	0.29536	0.20498	0.03313	0.01261

Table 28: Coefficients for the denominator $(D(s))$ of $H_{analog,BPF}(s)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	1	-1.61681	3.35166	-3.73352

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	4.97327	-3.73352	3.35166	-1.61681	1

Table 29: Coefficients for the numerator $(N(z))$ of $H_{discrete,BPF}(z)$

Degree	z^0	z^{-1}	z^{-2}	z^{-3}
Coefficient	2.7673	-3.42438	4.04586	-4.10673

Degree	z^{-4}	z^{-5}	z^{-6}	z^{-7}	z^{-8}
Coefficient	5.42296	-2.99498	1.80734	-1.07983	0.79016

Table 30: Coefficients for the denominator $(D(z))$ of $H_{discrete,BPF}(z)$

3.14 FIR Filter Parameters

Based on the required filter specifications, we define the following quantities:

$$A = -20 \log_{10} \delta = -20 \log_{10} 0.15 = 16.4782$$

Since $A < 21$, we take $\alpha = \beta = 0$, and therefore the Kaiser window will be rectangular in shape. The minimum width of the Kaiser window can be calculated using:

$$M \geq 1 + \frac{A - 8}{2.285 \Delta\omega_t} = 71.8627$$

where $\Delta\omega_t$ is the transition bandwidth, i.e., $5\pi/300$. We take the next odd integer value, i.e., 73. However, according to simulations, $M = 91$ is the minimum window width which properly meets specifications.

3.15 FIR Discrete Time Filter

The coefficients for the obtained FIR filter are given as follows:

```
Columns 1 through 18

  0.0000    0.0138    0.0041   -0.0019    0.0050   -0.0069   -0.0155    0.0014    0.0049   -0.0017    0.0125    0.0125   -0.0097   -0.0074   -0.0008   -0.0150   -0.0045    0.0190

Columns 19 through 36

  0.0072    0.0009    0.0134   -0.0078   -0.0264   -0.0019    0.0023   -0.0080    0.0225    0.0283   -0.0110   -0.0081    0.0000   -0.0373   -0.0199    0.0347    0.0144    0.0082

Columns 37 through 54

  0.0512   -0.0083   -0.0795   -0.0149   -0.0144   -0.0713    0.1213    0.2715   -0.0750    0.6167   -0.0750    0.2715    0.1213   -0.0713   -0.0144   -0.0149   -0.0795   -0.0083

Columns 55 through 72

  0.0512    0.0082    0.0144    0.0347   -0.0199   -0.0373    0.0000   -0.0081   -0.0110    0.0283    0.0225   -0.0080    0.0023   -0.0019   -0.0264   -0.0078    0.0134    0.0009

Columns 73 through 90

  0.0072    0.0190   -0.0045   -0.0150   -0.0008   -0.0074   -0.0097    0.0125    0.0125   -0.0017    0.0049    0.0014   -0.0155   -0.0069    0.0050   -0.0019    0.0041    0.0138

Column 91

  0.0000
```

Figure 8: Coefficients of $H_{discrete,BSF}(z)$

4 Cascading the two filters

4.1 Butterworth Cascaded Filter

The discrete-time transfer function after cascading the bandpass and bandstop filters is given as follows:

$$H_{discrete,cascade}(z) = H_{discrete,BPF}(z) \times H_{discrete,BSF}(z)$$

```
  1.0e+10 *

 Columns 1 through 17

   0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0002    0.0002   -0.0000

 Columns 18 through 34

  -0.0006    0.0013   -0.0012   -0.0010    0.0050   -0.0073    0.0023    0.0119   -0.0268    0.0236    0.0122   -0.0670    0.0918   -0.0318   -0.1104    0.2354   -0.1937

 Columns 35 through 51

  -0.0772    0.4289   -0.5425    0.1792    0.5280   -1.0397    0.7845    0.2827   -1.4459    1.6712   -0.4960   -1.3829    2.4770   -1.6954   -0.6014    2.6935   -2.8283

 Columns 52 through 68

   0.7319    2.0371   -3.2965    2.0371    0.7319   -2.8283    2.6935   -0.6014   -1.6954    2.4770   -1.3829   -0.4960    1.6712   -1.4459    0.2827    0.7845   -1.0397

 Columns 69 through 85

   0.5280    0.1792   -0.5425    0.4289   -0.0772   -0.1937    0.2354   -0.1104   -0.0318    0.0918   -0.0670    0.0122    0.0236   -0.0268    0.0119    0.0023   -0.0073

 Columns 86 through 102

   0.0050   -0.0010   -0.0012    0.0013   -0.0006   -0.0000    0.0002   -0.0002    0.0000    0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000   -0.0000    0.0000

 Columns 103 through 107

   0.0000   -0.0000    0.0000   -0.0000    0.0000
```

Figure 9: Coefficients for the numerator $(N(z))$ of $H_{discrete,cascade}(z)$

```
  1.0e+16 *

 Columns 1 through 17

   0.0000   -0.0000    0.0001   -0.0005    0.0020   -0.0061    0.0156   -0.0353    0.0721   -0.1355    0.2371   -0.3899    0.6072   -0.9002    1.2765   -1.7375    2.2771

 Columns 18 through 34

  -2.8807    3.5253   -4.1810    4.8131   -5.3856    5.8644   -6.2209    6.4346   -6.4951    6.4026   -6.1677    5.8093   -5.3528    4.8274   -4.2626    3.6867   -3.1242

 Columns 35 through 51

   2.5948   -2.1127    1.6868   -1.3208    1.0145   -0.7645    0.5652   -0.4101    0.2920   -0.2041    0.1400   -0.0943    0.0623   -0.0404    0.0257   -0.0161    0.0099

 Columns 52 through 68

  -0.0059    0.0035   -0.0020    0.0012   -0.0006    0.0004   -0.0002    0.0001   -0.0001    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000

 Columns 69 through 85

   0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000

 Columns 86 through 102

  -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000

 Columns 103 through 107

  -0.0000   -0.0000    0.0000   -0.0000    0.0000
```

Figure 10: Coefficients for the denominator $(D(z))$ of $H_{discrete,cascade}(z)$

4.2 Chebyshev Cascaded Filter

The discrete-time transfer function after cascading the bandpass and bandstop filters is given as follows:

$$H_{discrete,cascade}(z) = H_{discrete,BPF}(z) \times H_{discrete,BSF}(z)$$

```
  Columns 1 through 18

   -1.0000    4.1119   -6.3971   -0.6198   25.3058  -53.0785   38.1022   53.1932 -179.4541  202.7299   -8.4281 -319.1459  496.8299 -267.5239 -281.3893  705.5142 -601.0409         0

  Columns 19 through 35

  601.0409 -705.5142  281.3893  267.5239 -496.8299  319.1459    8.4281 -202.7299  179.4541  -53.1932  -38.1022   53.0785  -25.3058    0.6198    6.3971   -4.1119    1.0000
```

Figure 11: Coefficients for the numerator $(N(z))$ of $H_{discrete,cascade}(z)$

```
   1.0e+06 *

  Columns 1 through 18

    0.0290   -0.1550    0.4051   -0.7560    1.2903   -2.1164    3.1020   -4.0656    5.0816   -6.1263    6.8663   -7.1771    7.2450   -7.0450    6.4306   -5.5598    4.6712   -3.7778

  Columns 19 through 35

    2.8867   -2.1248    1.5480   -1.1042    0.7658   -0.5359    0.3866   -0.2767    0.1927   -0.1346    0.0930   -0.0584    0.0329   -0.0179    0.0091   -0.0034    0.0006
```

Figure 12: Coefficients for the denominator $(D(z))$ of $H_{discrete,cascade}(z)$

4.3 Elliptical Cascaded Filter

The discrete-time transfer function after cascading the bandpass and bandstop filters is given as follows:

$$H_{discrete,cascade}(z) = H_{discrete,BPF}(z) \times H_{discrete,BSF}(z)$$

```
1.0000   -2.2212   2.3265   -2.3946   3.3767   -3.6637   3.2360   -3.9139   4.8196   -3.9139   3.2360   -3.6637   3.3767   -2.3946   2.3265   -2.2212   1.0000
```

Figure 13: Coefficients for the numerator $(N(z))$ of $H_{discrete,cascade}(z)$

```
7.6580  -18.1035   20.3297  -24.0754   40.3011  -45.9150   37.2601  -37.0526   41.2740  -30.2627   17.9183  -14.7576   11.2734   -4.7200   1.9443   -1.4456   0.6243
```

Figure 14: Coefficients for the denominator $(D(z))$ of $H_{discrete,cascade}(z)$

4.4 FIR Cascaded Filter

The discrete-time transfer function after cascading the bandpass and bandstop filters is given as follows:

$$H_{discrete,cascade}(z) = H_{discrete,BPF}(z) \times H_{discrete,BSF}(z)$$

```
Columns 1 through 18

  0.0000    0.0000    0.0002    0.0001    0.0000    0.0002   -0.0001   -0.0003   -0.0000   -0.0003   -0.0002    0.0002    0.0000    0.0001    0.0002    0.0000   -0.0000   -0.0000

Columns 19 through 36

  0.0002    0.0002    0.0000    0.0004   -0.0001   -0.0005   -0.0000   -0.0005   -0.0005    0.0003    0.0000    0.0001    0.0005    0.0001   -0.0000    0.0000    0.0003    0.0004

Columns 37 through 54

  0.0001    0.0010    0.0001   -0.0016   -0.0002   -0.0020   -0.0033    0.0013    0.0001    0.0004    0.0217    0.0073    0.0000    0.0145   -0.0135   -0.0165   -0.0001   -0.0119

Columns 55 through 72

 -0.0036    0.0034    0.0001   -0.0032   -0.0138    0.0033    0.0000   -0.0052    0.0372    0.0188   -0.0000    0.0307   -0.0155   -0.0341   -0.0000   -0.0266   -0.0160    0.0119

Columns 73 through 90

  0.0000   -0.0004   -0.0157    0.0000    0.0000   -0.0193    0.0691    0.0547   -0.0000    0.0946   -0.0165   -0.1225   -0.0000   -0.1237   -0.1426    0.0922    0.0000    0.0342

Columns 91 through 108

  0.2336    0.0342    0.0000    0.0922   -0.1426   -0.1237   -0.0000   -0.1225   -0.0165    0.0946   -0.0000    0.0547    0.0691   -0.0193    0.0000    0.0000   -0.0157   -0.0004

Columns 109 through 126

  0.0000    0.0119   -0.0160   -0.0266   -0.0000   -0.0341   -0.0155    0.0307   -0.0000    0.0188    0.0372   -0.0052    0.0000    0.0033   -0.0138   -0.0032    0.0001    0.0034

Columns 127 through 144

 -0.0036   -0.0119   -0.0001   -0.0165   -0.0135    0.0145    0.0000    0.0073    0.0217    0.0004    0.0001    0.0013   -0.0033   -0.0020   -0.0002   -0.0016    0.0001    0.0010

Columns 145 through 162

  0.0001    0.0004    0.0003    0.0000   -0.0000    0.0001    0.0005    0.0001    0.0000    0.0003   -0.0005   -0.0005   -0.0000   -0.0005   -0.0001    0.0004    0.0000    0.0002

Columns 163 through 180

  0.0002   -0.0000   -0.0000    0.0000    0.0002    0.0001    0.0000    0.0002   -0.0002   -0.0003   -0.0000   -0.0003   -0.0001    0.0002    0.0000    0.0001    0.0002    0.0000

Column 181

  0.0000
```

Figure 15: Coefficients of $H_{discrete,cascade}(z)$

5 MATLAB Simulations

5.1 Acknowledgement

The report format and the MATLAB codes are inspired by the Ashwin Bhat's previous year submission (available on MSTeams Class Resources).

5.2 Butterworth Bandpass Filter

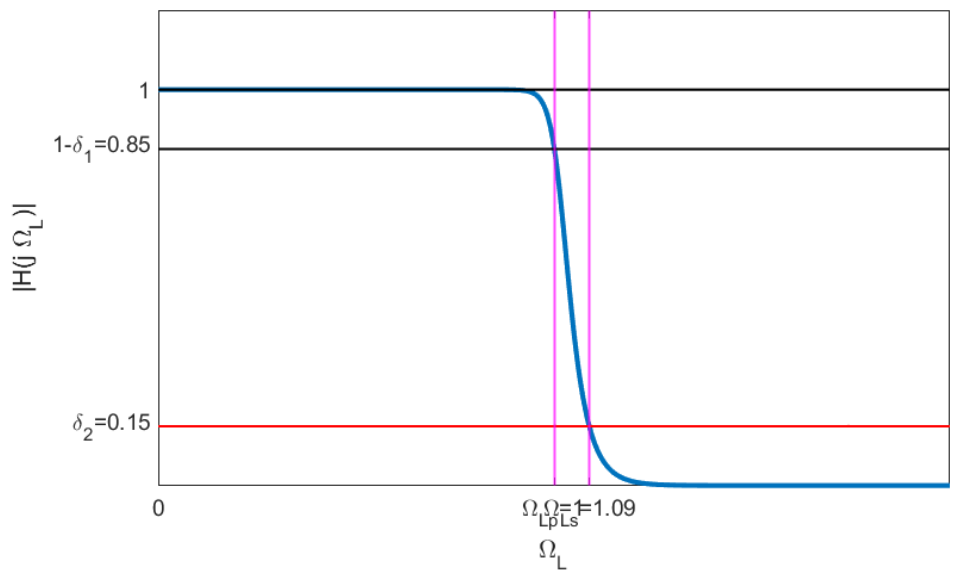

Figure 16: Lowpass Analog Filter Response for the Bandpass Filter

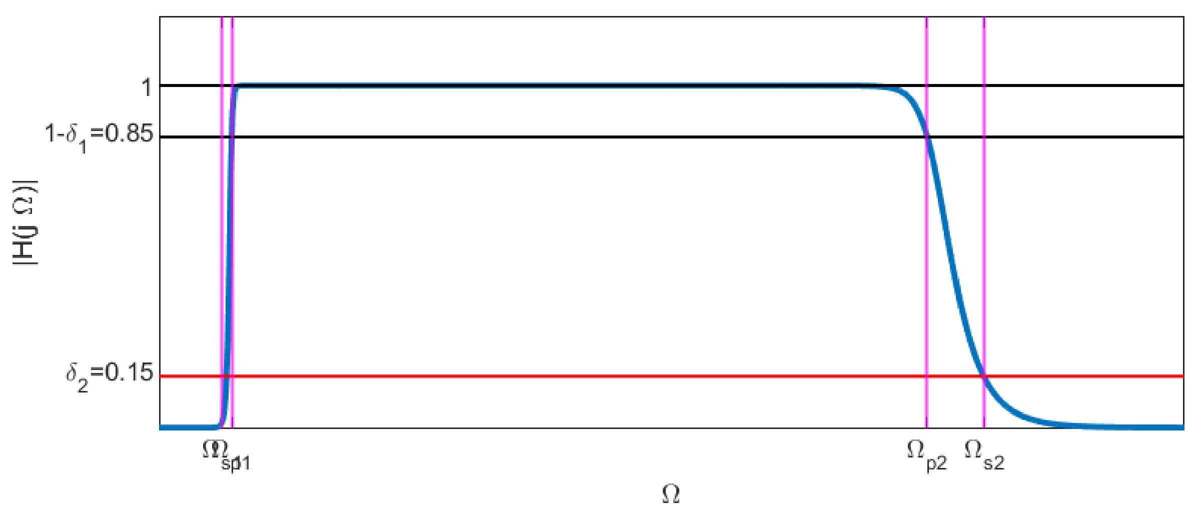

Figure 17: Magnitude Response for the Bandpass Analog Filter

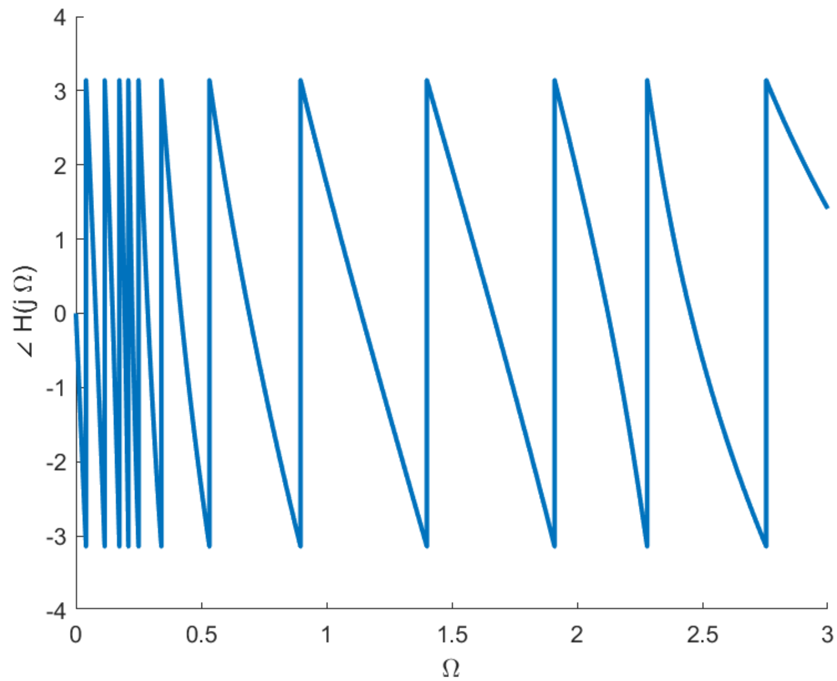

Figure 18: Phase Response for the Bandpass Analog Filter

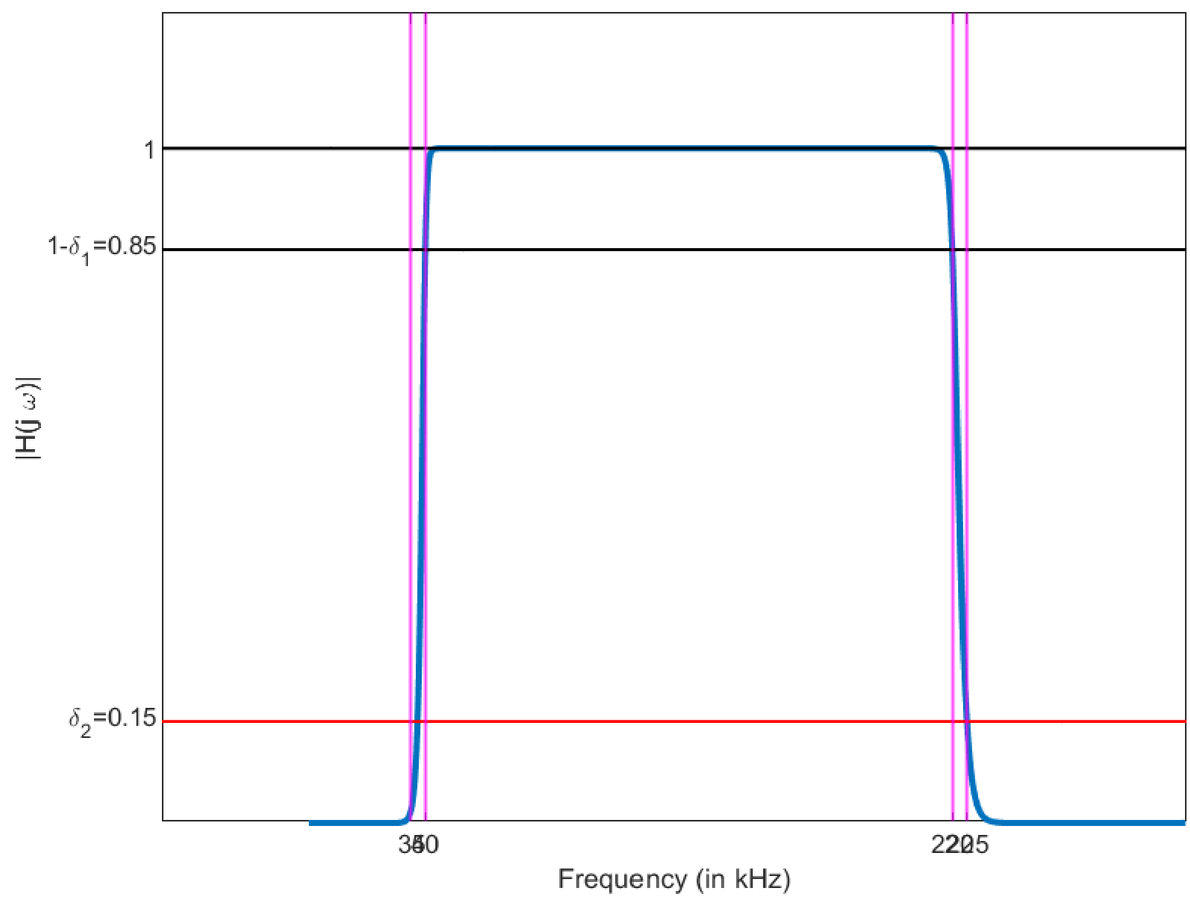

Figure 19: Bandpass Discrete-time Filter Response

5.3 Butterworth Bandstop Filter

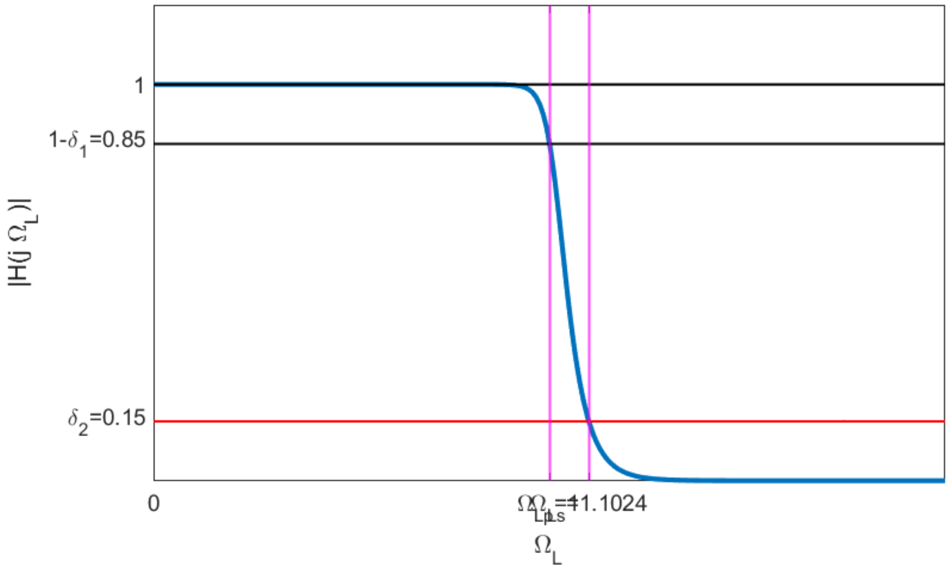

Figure 20: Lowpass Analog Filter Response for the Bandstop Filter

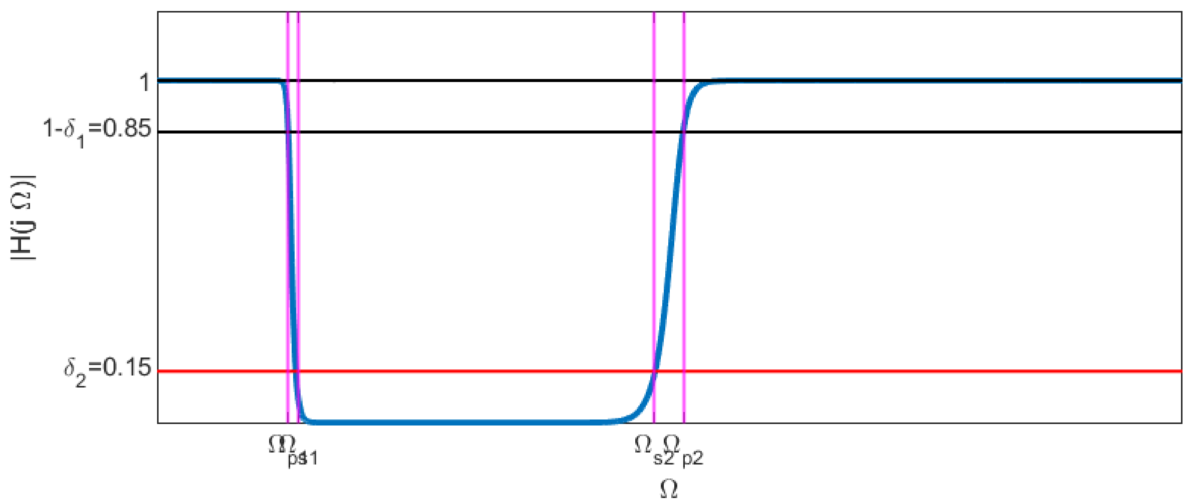

Figure 21: Magnitude Response for the Bandstop Filter

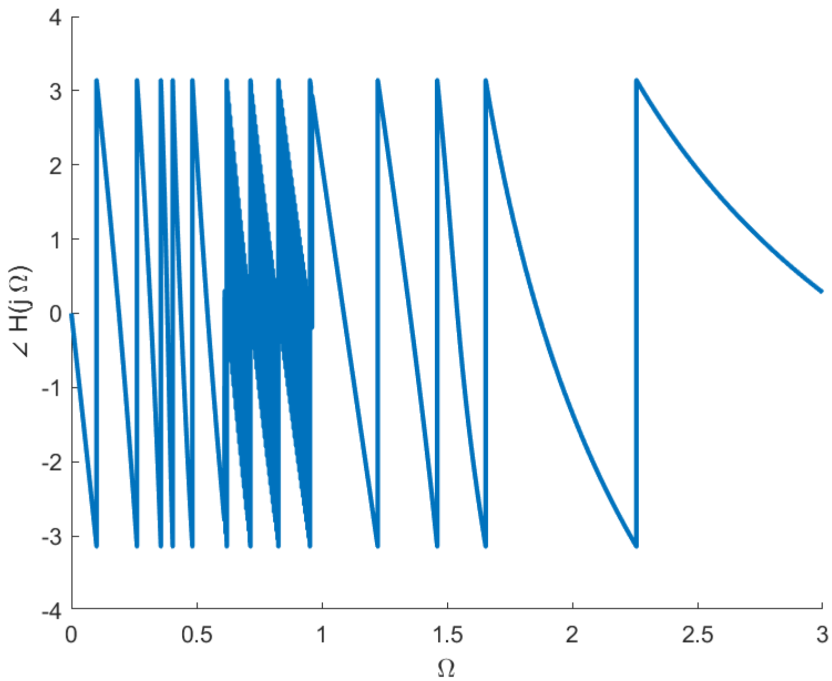

Figure 22: Phase Response for the Bandstop Filter

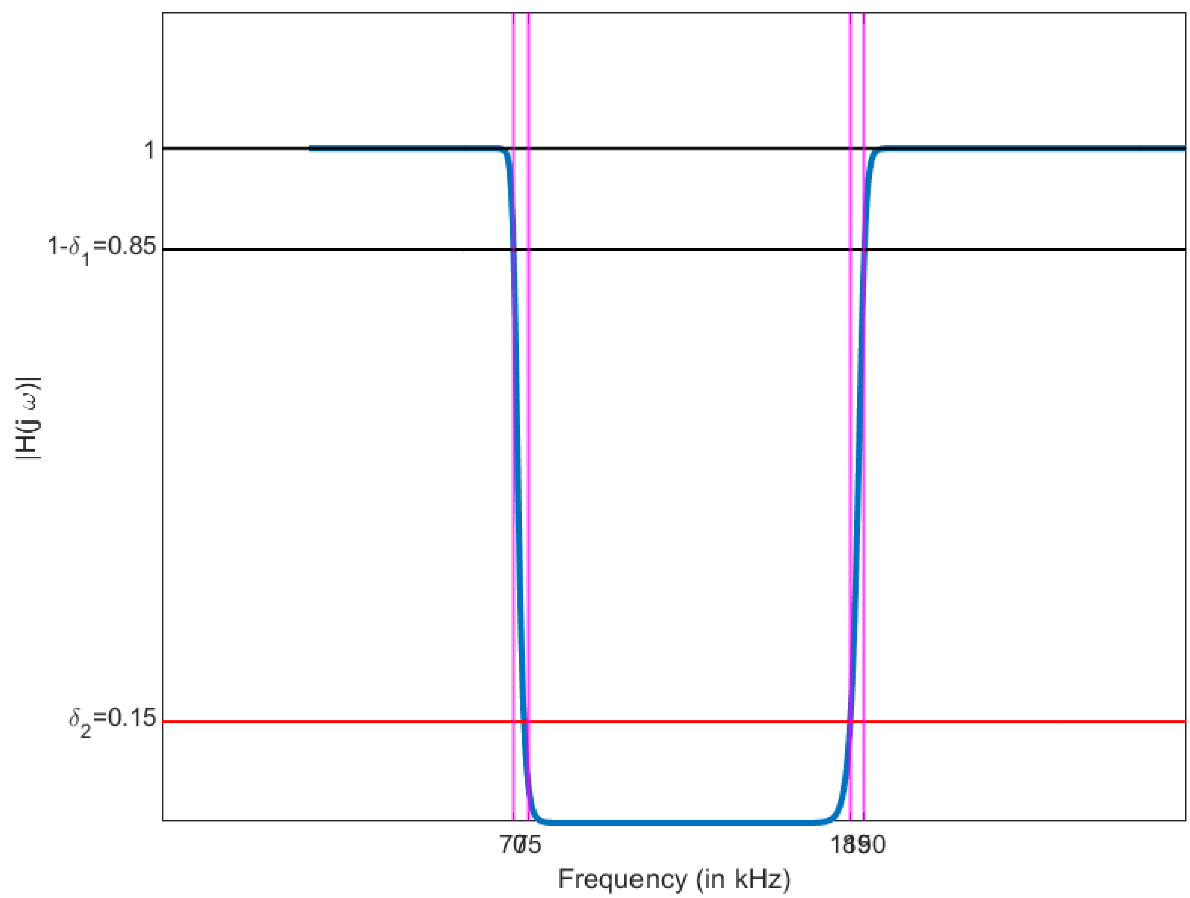

Figure 23: Bandstop Discrete-time Filter Response

5.4 Butterworth Cascaded filter

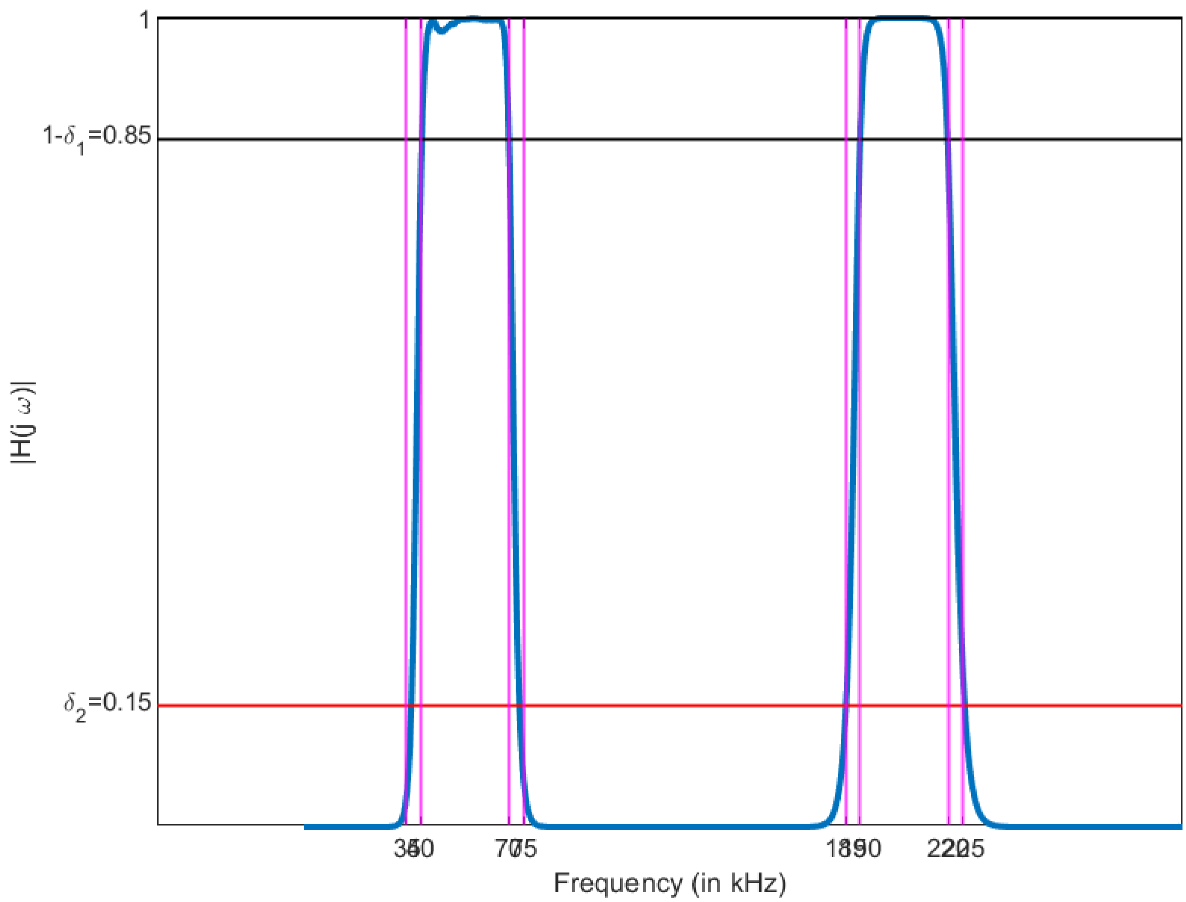

Figure 24: Cascade Discrete-time Filter Magnitude Response

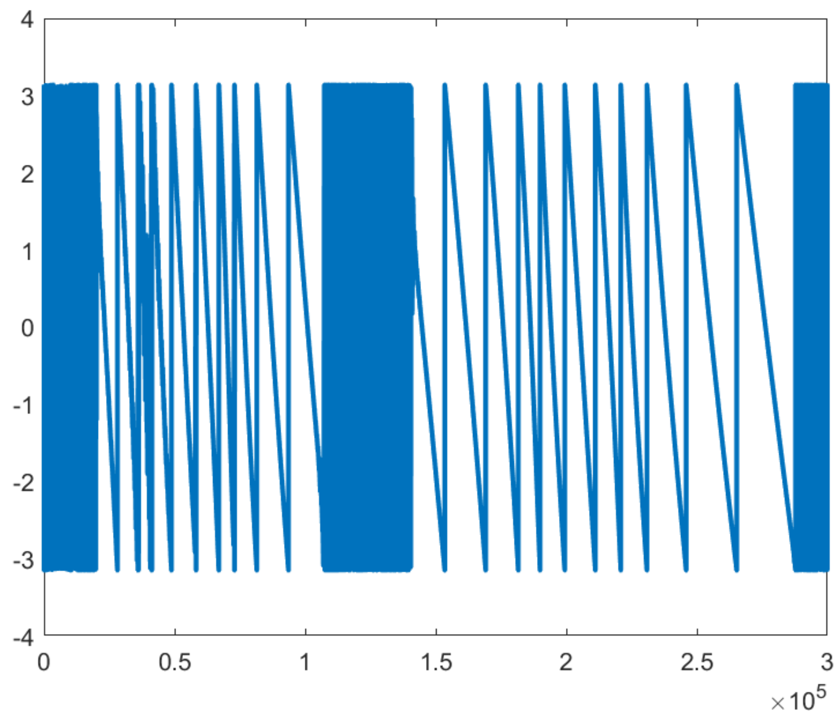

Figure 25: Cascade Discrete-time Filter Filter Response

5.5 Chebyshev Bandpass Filter

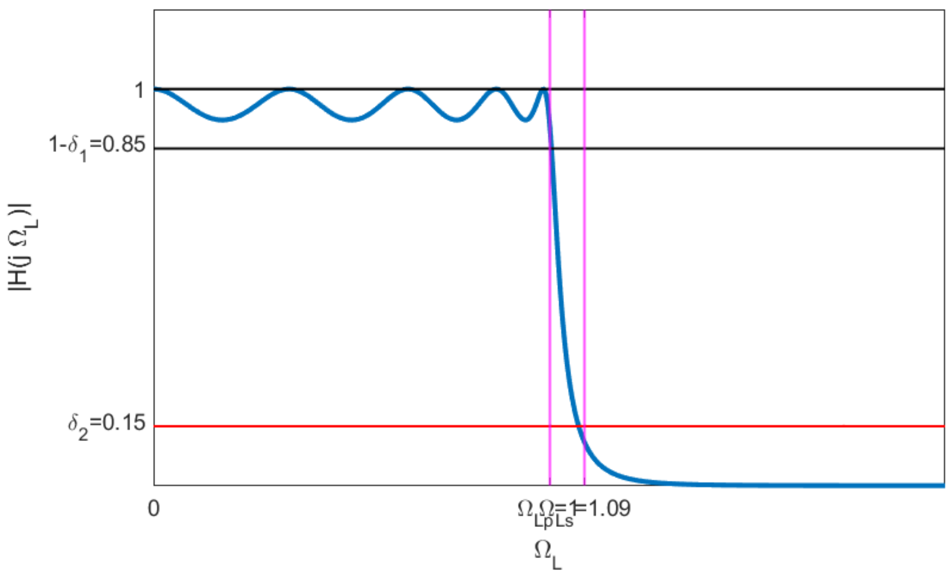

Figure 26: Lowpass Analog Filter Response for the Bandpass Filter

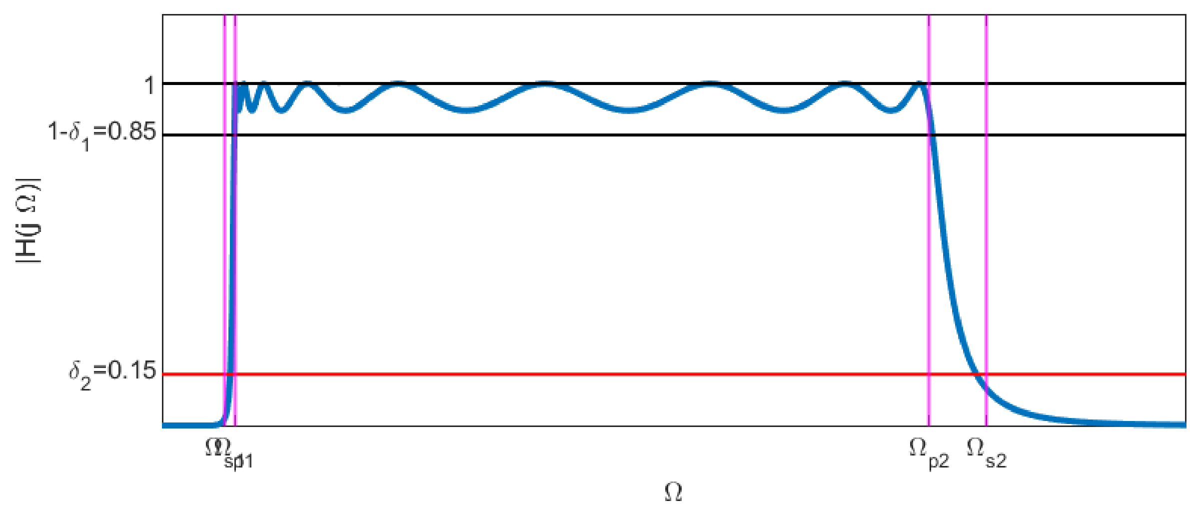

Figure 27: Magnitude Response for the Bandpass Analog Filter

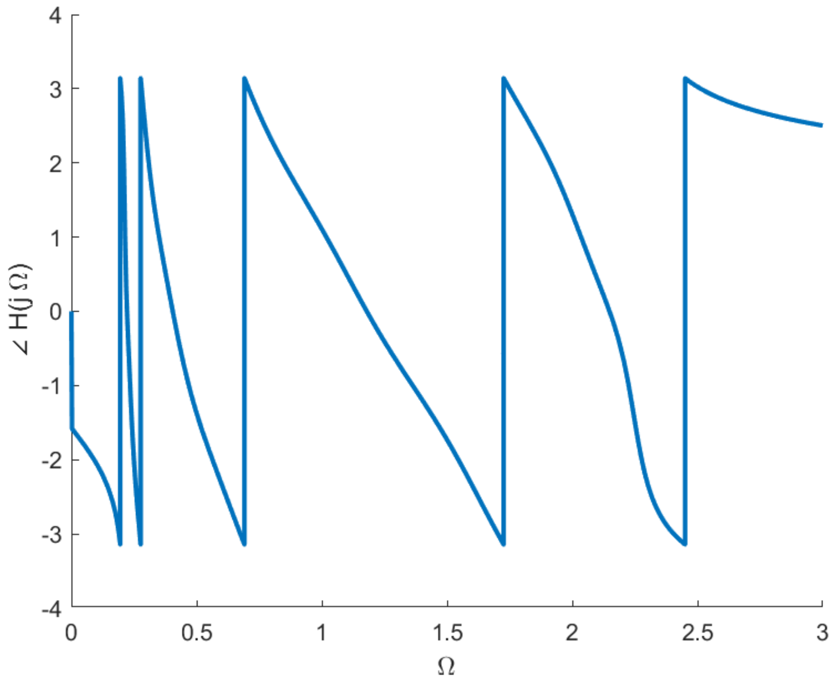

Figure 28: Phase Response for the Bandpass Analog Filter

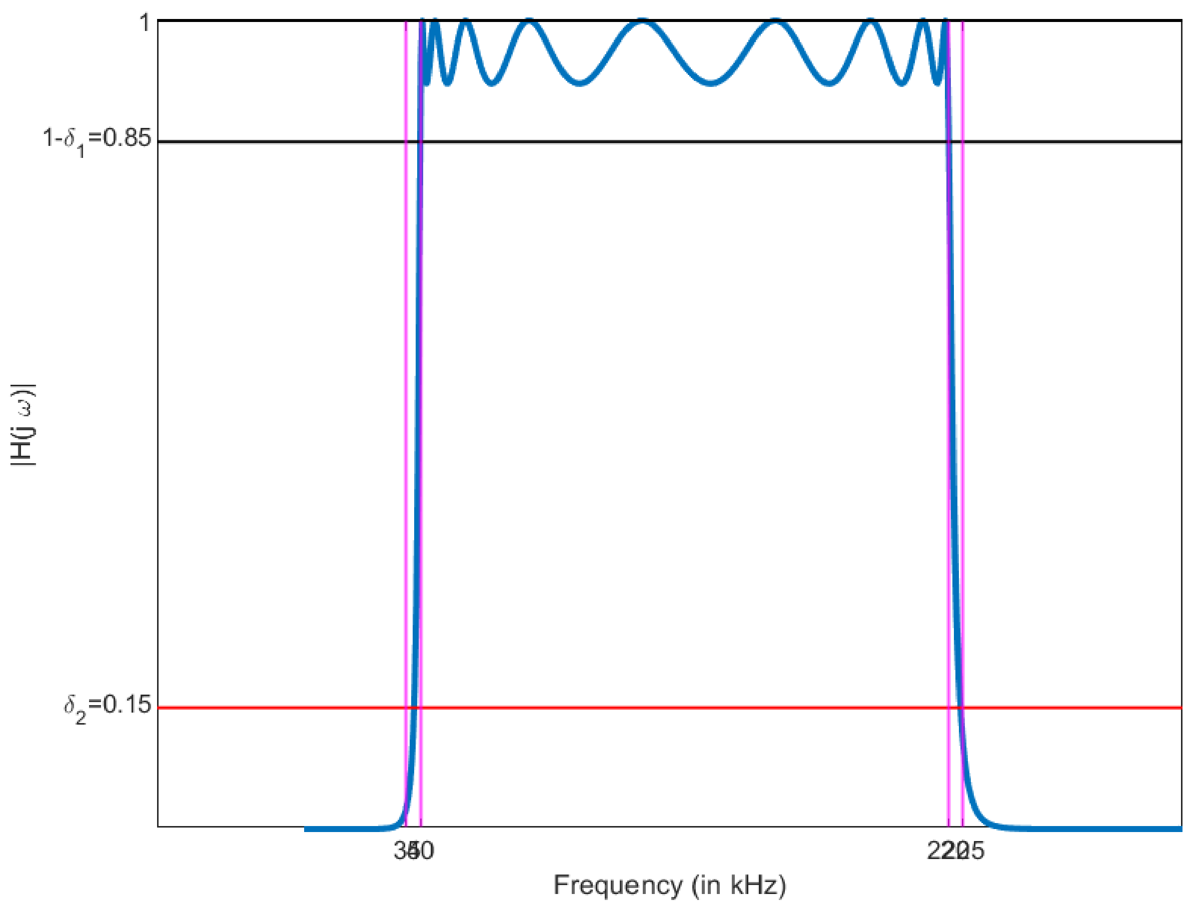

Figure 29: Bandpass Discrete-time Filter Response

5.6 Chebyshev Bandstop Filter

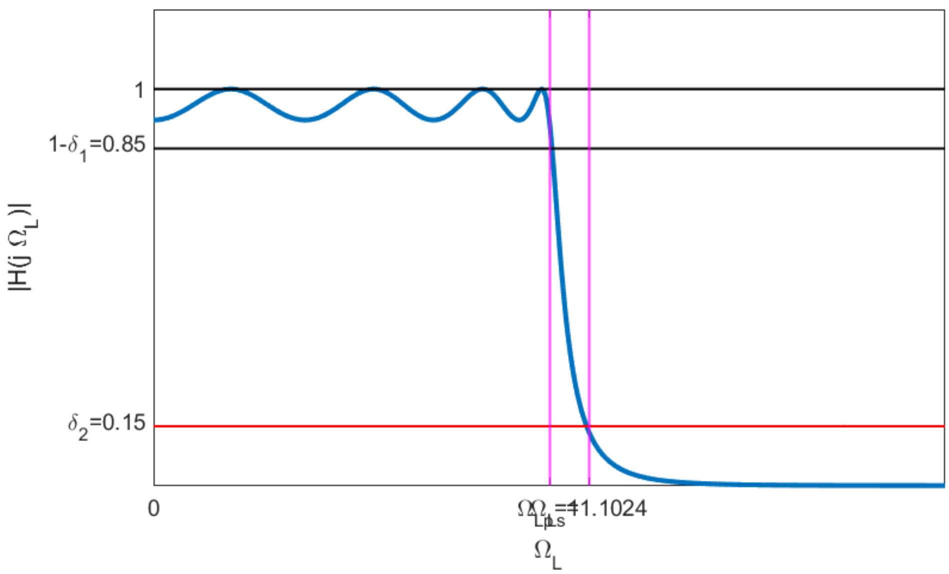

Figure 30: Lowpass Analog Filter Response for the Bandstop Filter

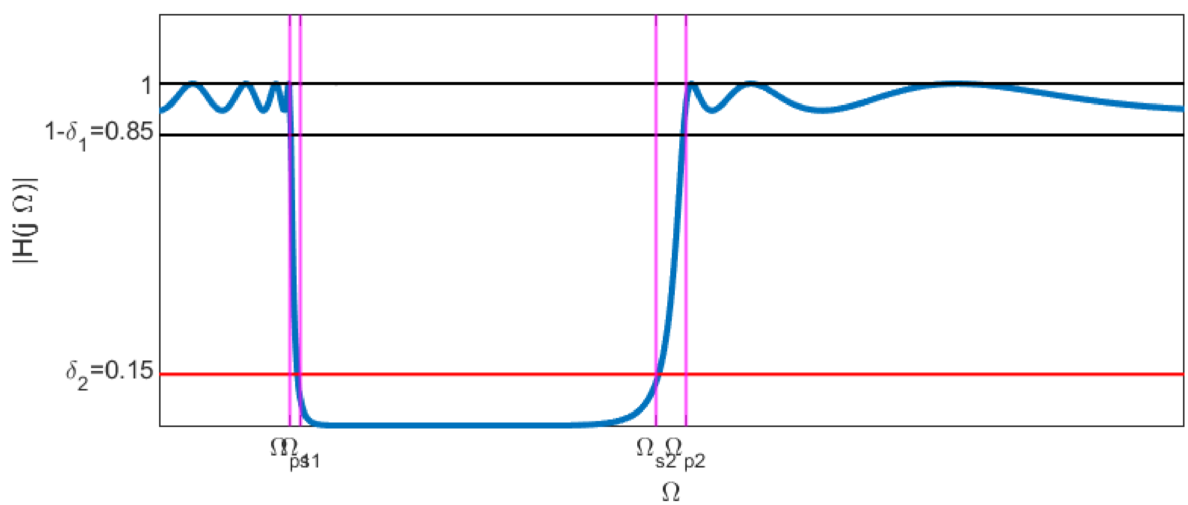

Figure 31: Magnitude Response for the Bandstop Filter

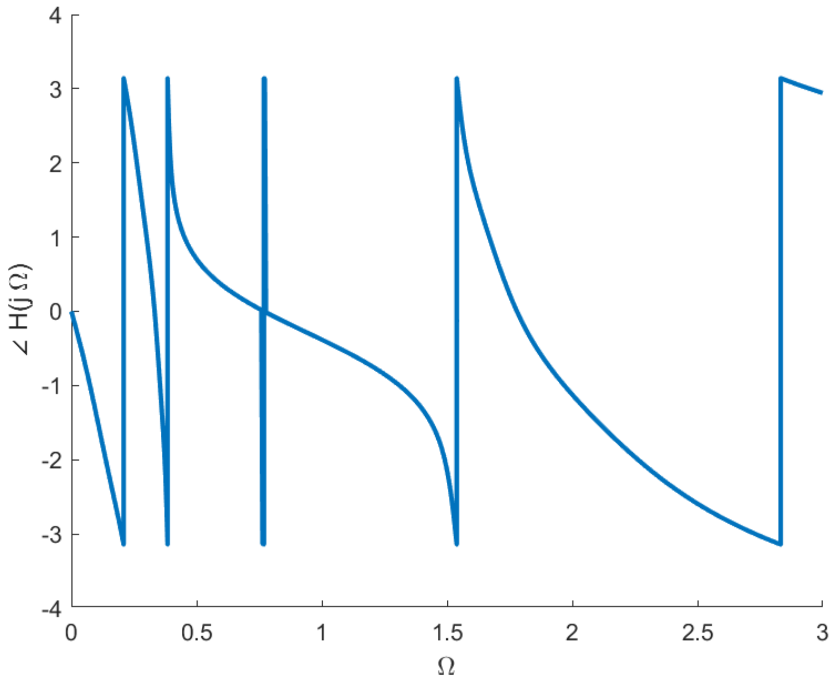

Figure 32: Phase Response for the Bandstop Filter

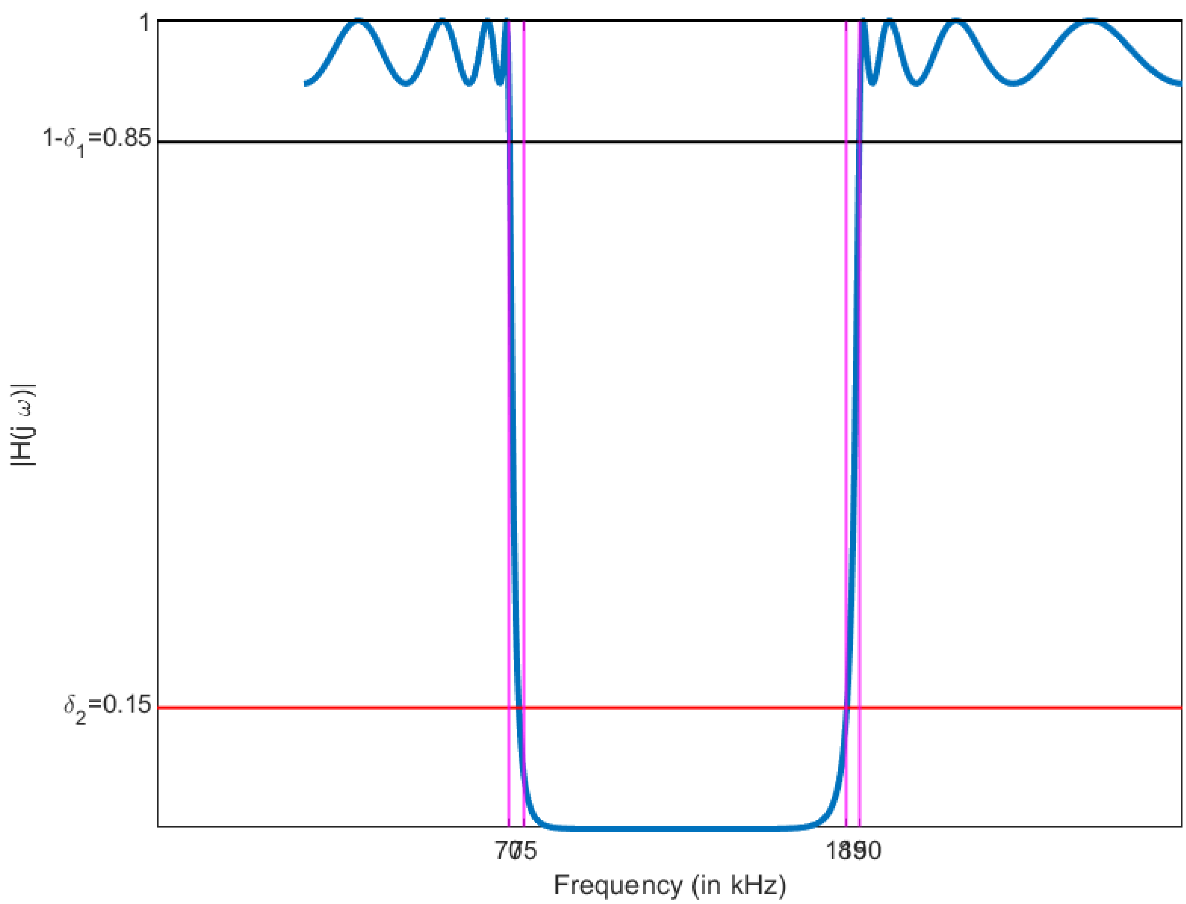

Figure 33: Bandstop Discrete-time Filter Response

5.7 Chebyshev Cascaded filter

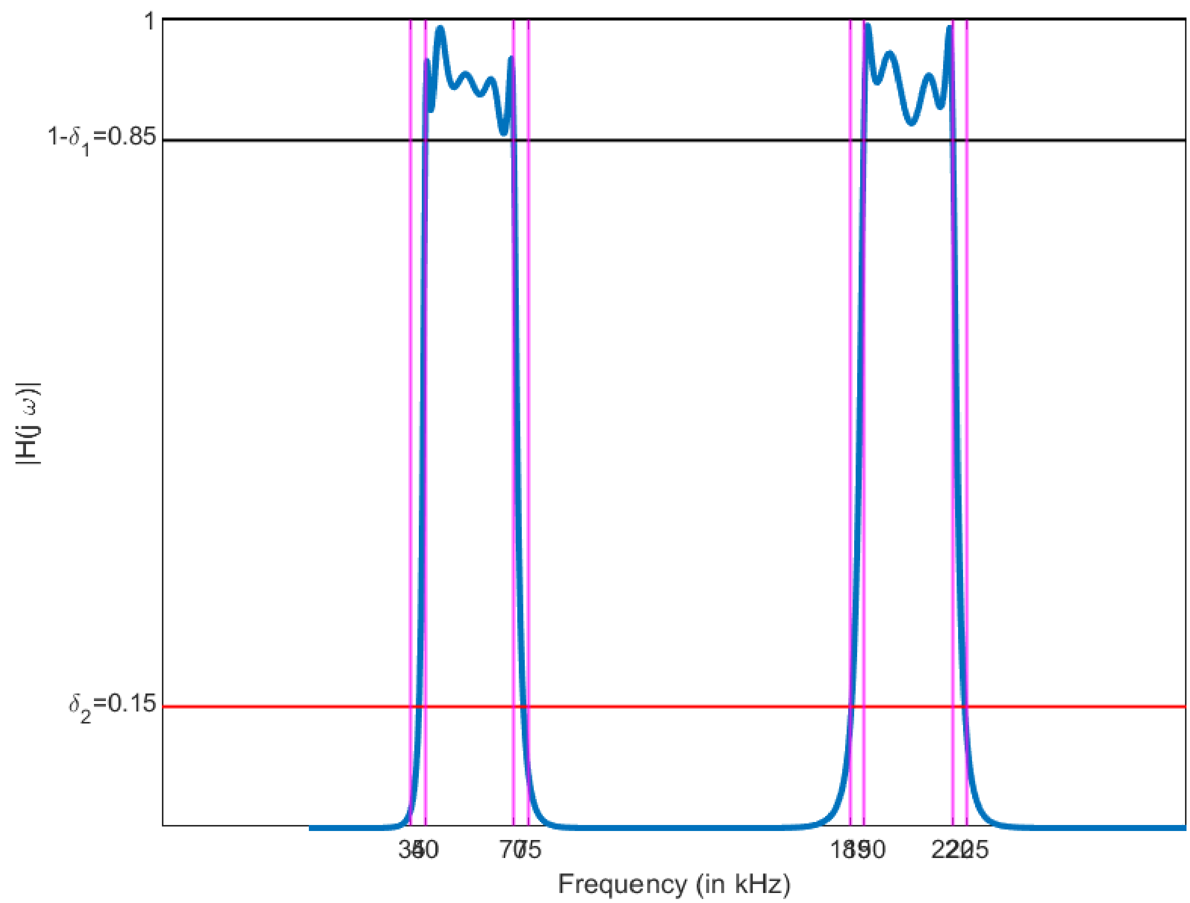

Figure 34: Cascade Discrete-time Filter Magnitude Response

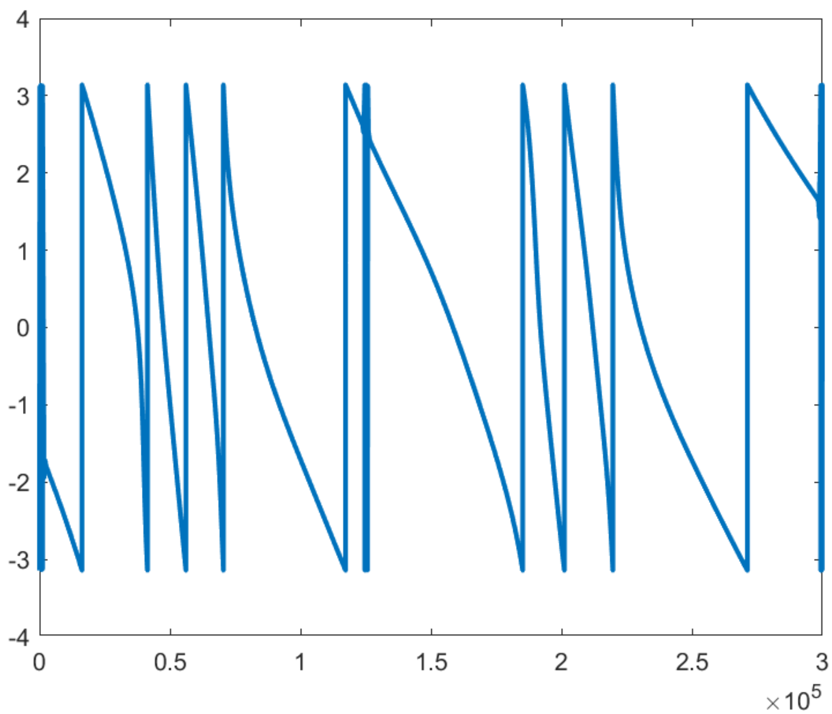

Figure 35: Cascade Discrete-time Filter Filter Response

5.8 Elliptical Bandpass Filter

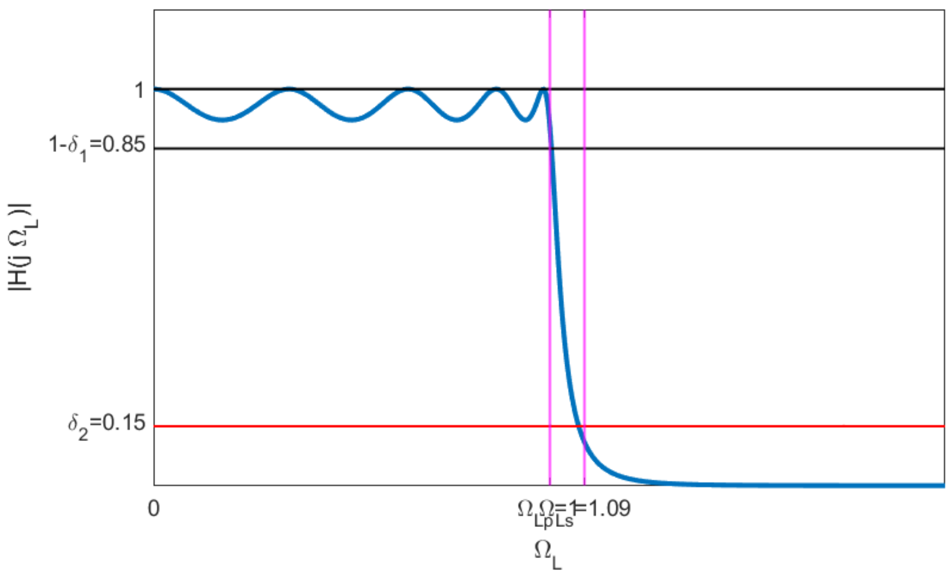

Figure 36: Lowpass Analog Filter Response for the Bandpass Filter

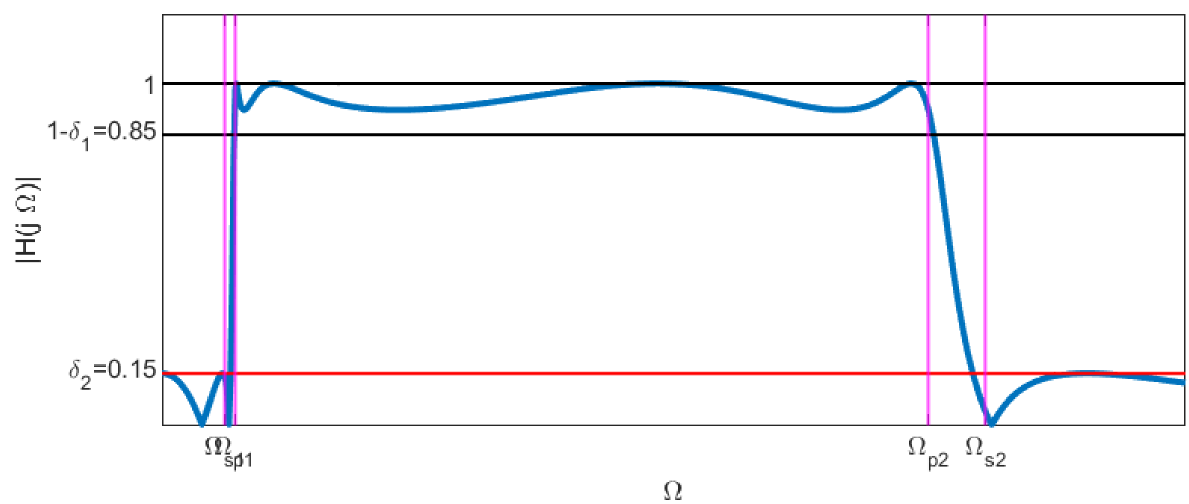

Figure 37: Magnitude Response for the Bandpass Analog Filter

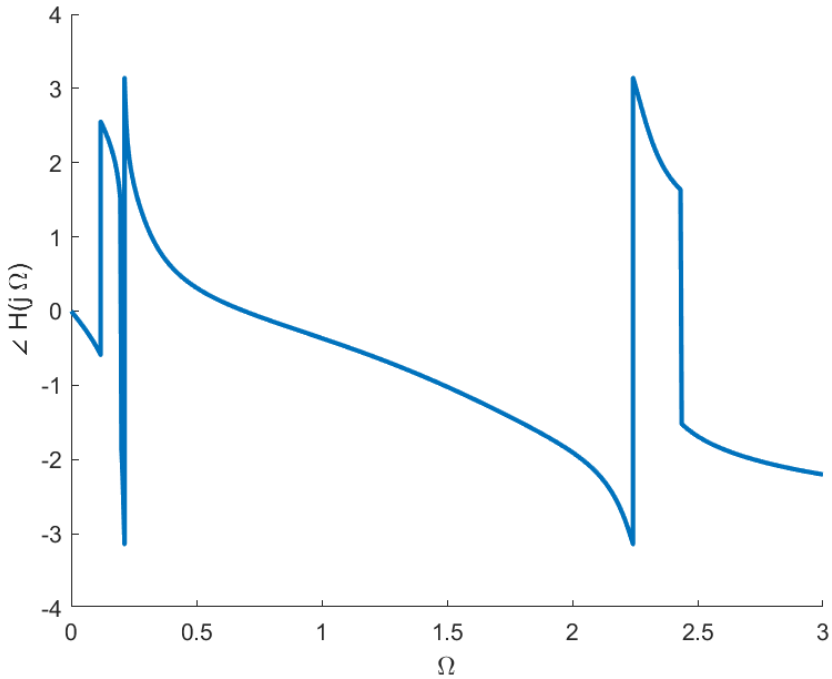

Figure 38: Phase Response for the Bandpass Analog Filter

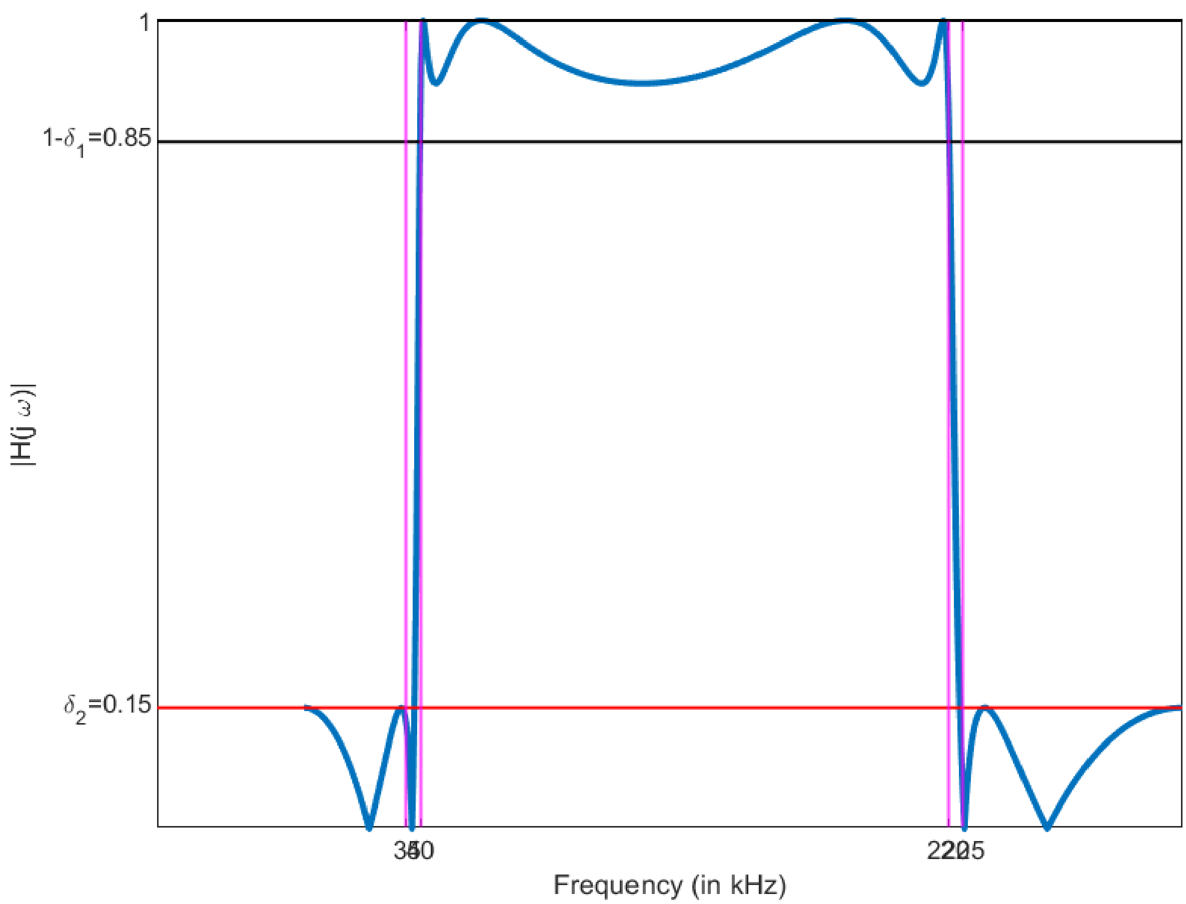

Figure 39: Bandpass Discrete-time Filter Response

5.9 Elliptical Bandstop Filter

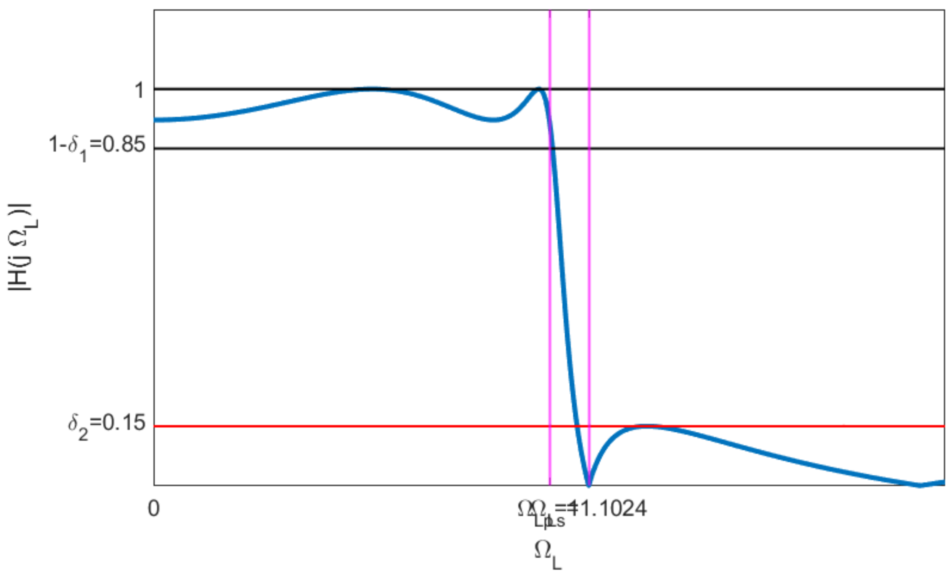

Figure 40: Lowpass Analog Filter Response for the Bandstop Filter

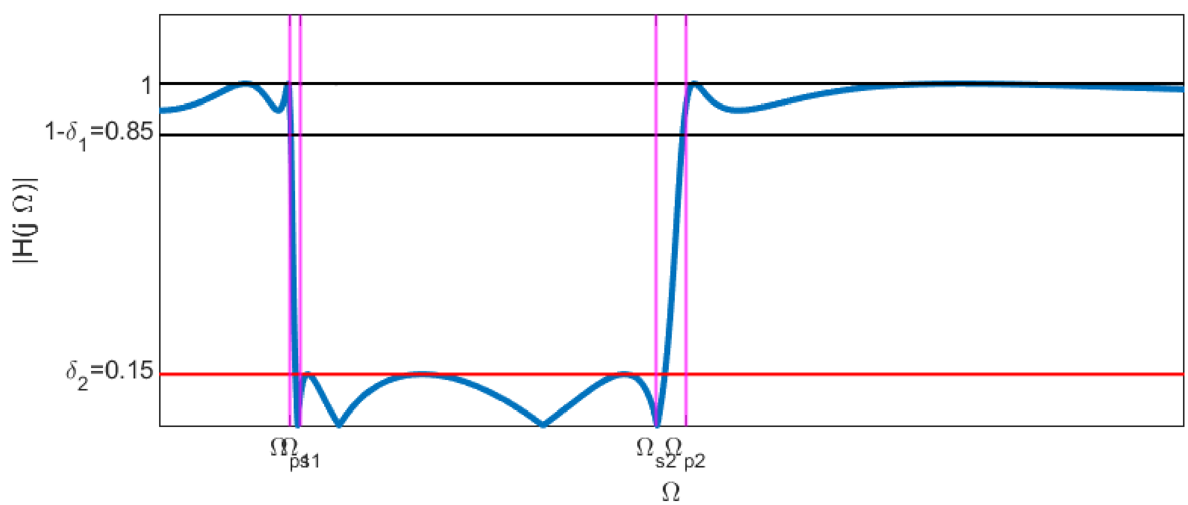

Figure 41: Magnitude Response for the Bandstop Filter

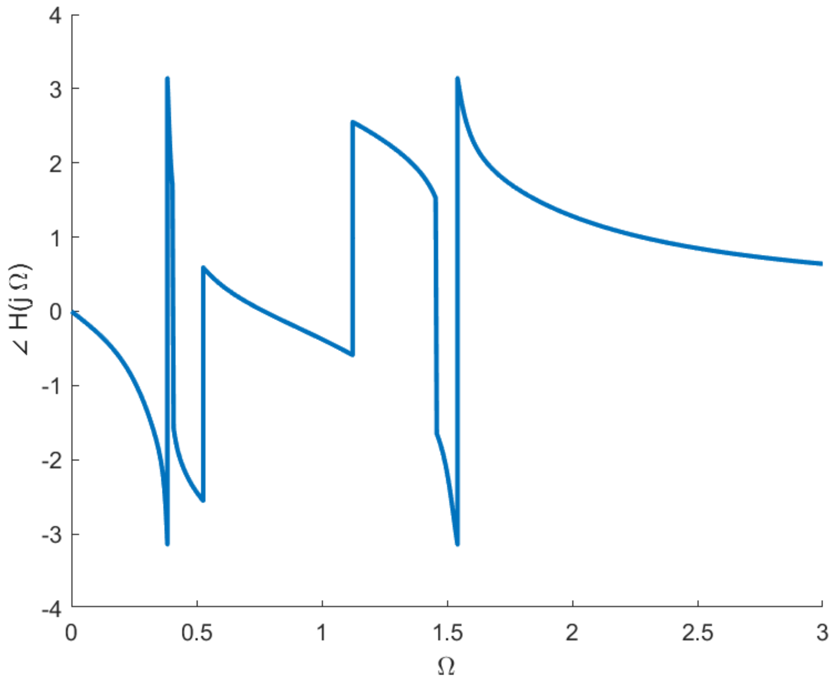

Figure 42: Phase Response for the Bandstop Filter

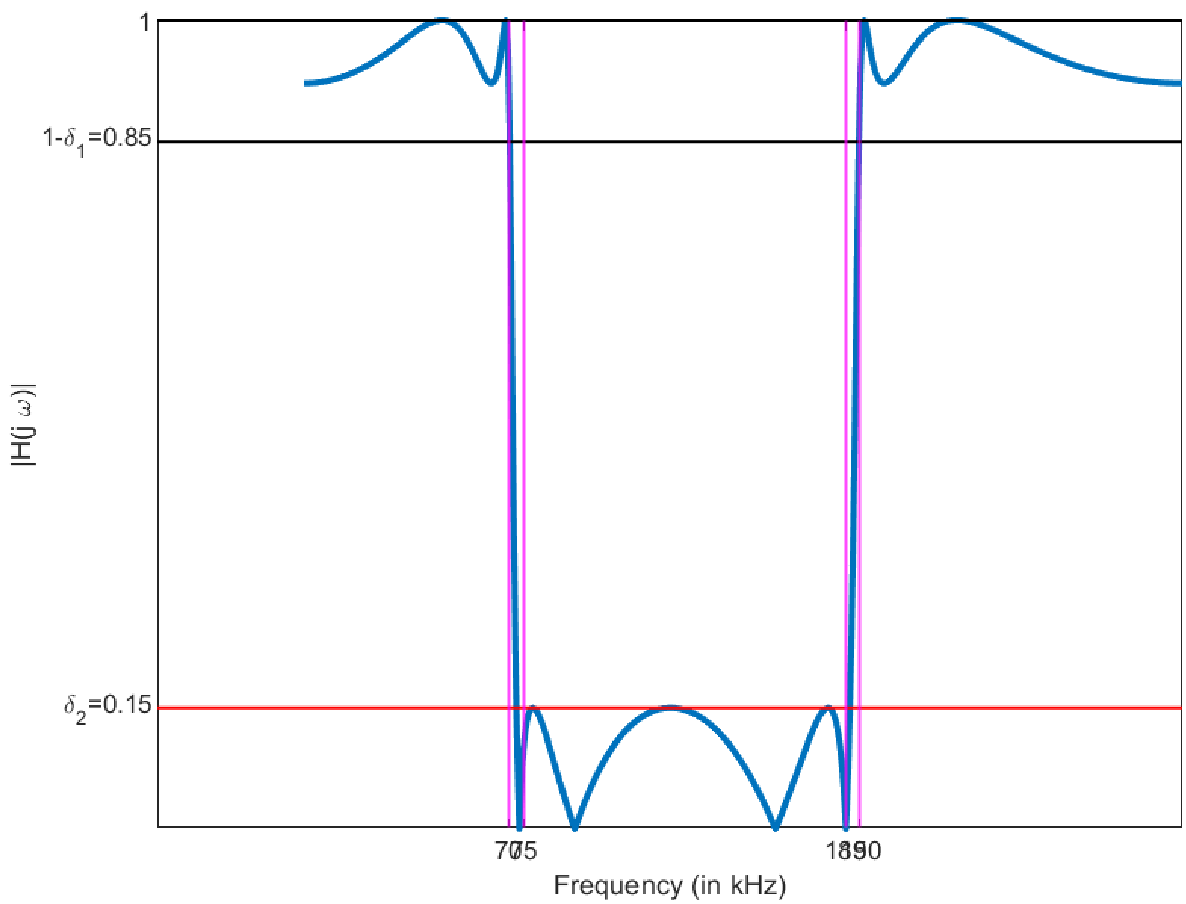

Figure 43: Bandstop Discrete-time Filter Response

5.10 Elliptical Cascaded filter

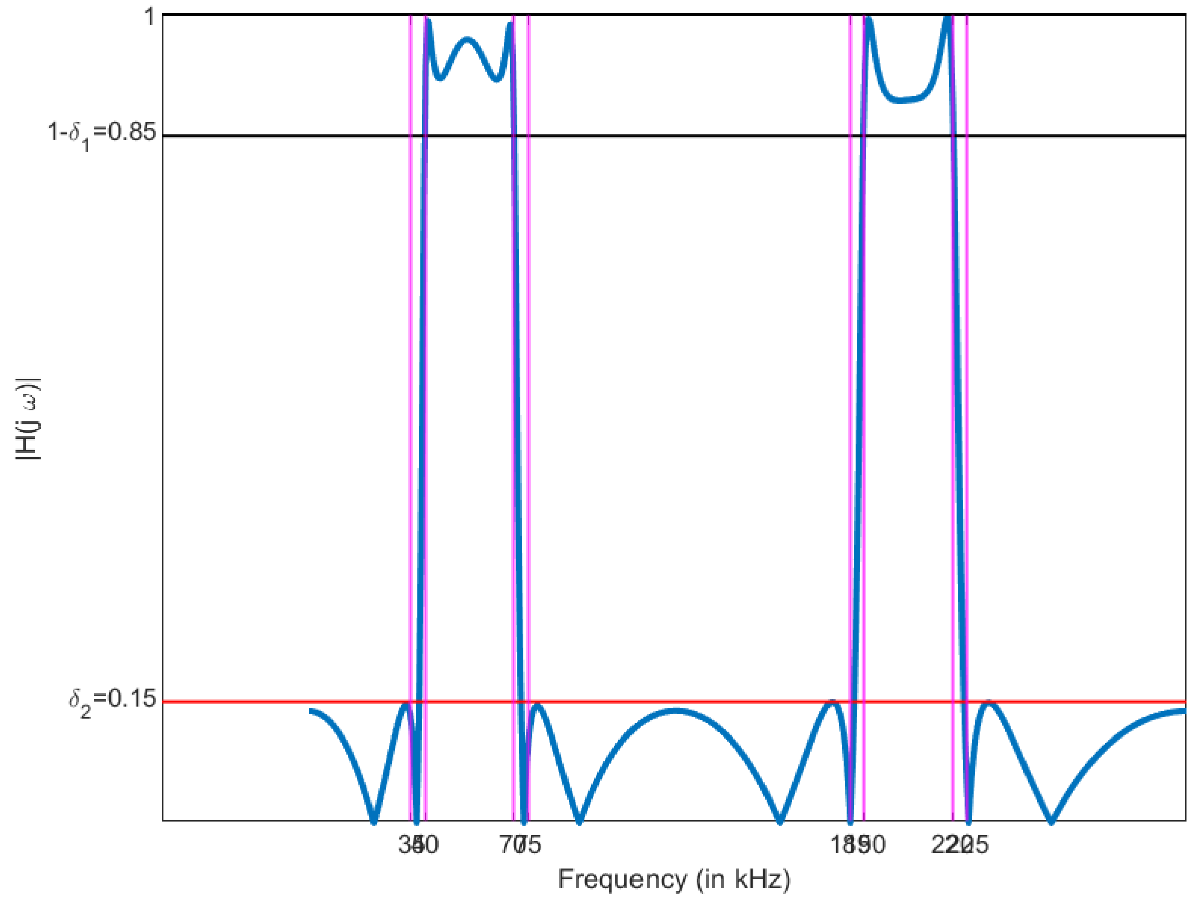

Figure 44: Cascade Discrete-time Filter Magnitude Response

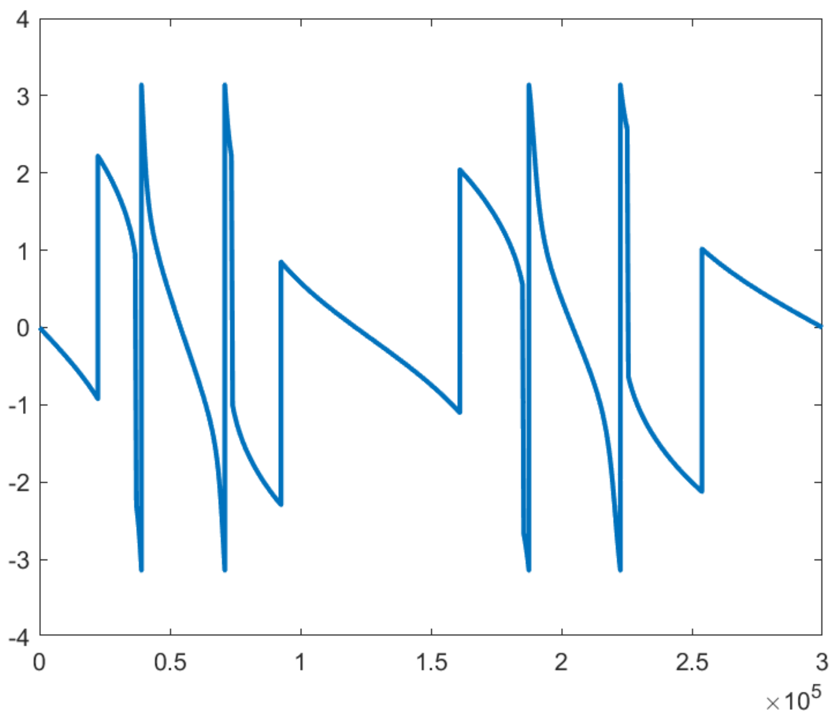

Figure 45: Cascade Discrete-time Filter Filter Response

5.11 FIR Bandpass Filter

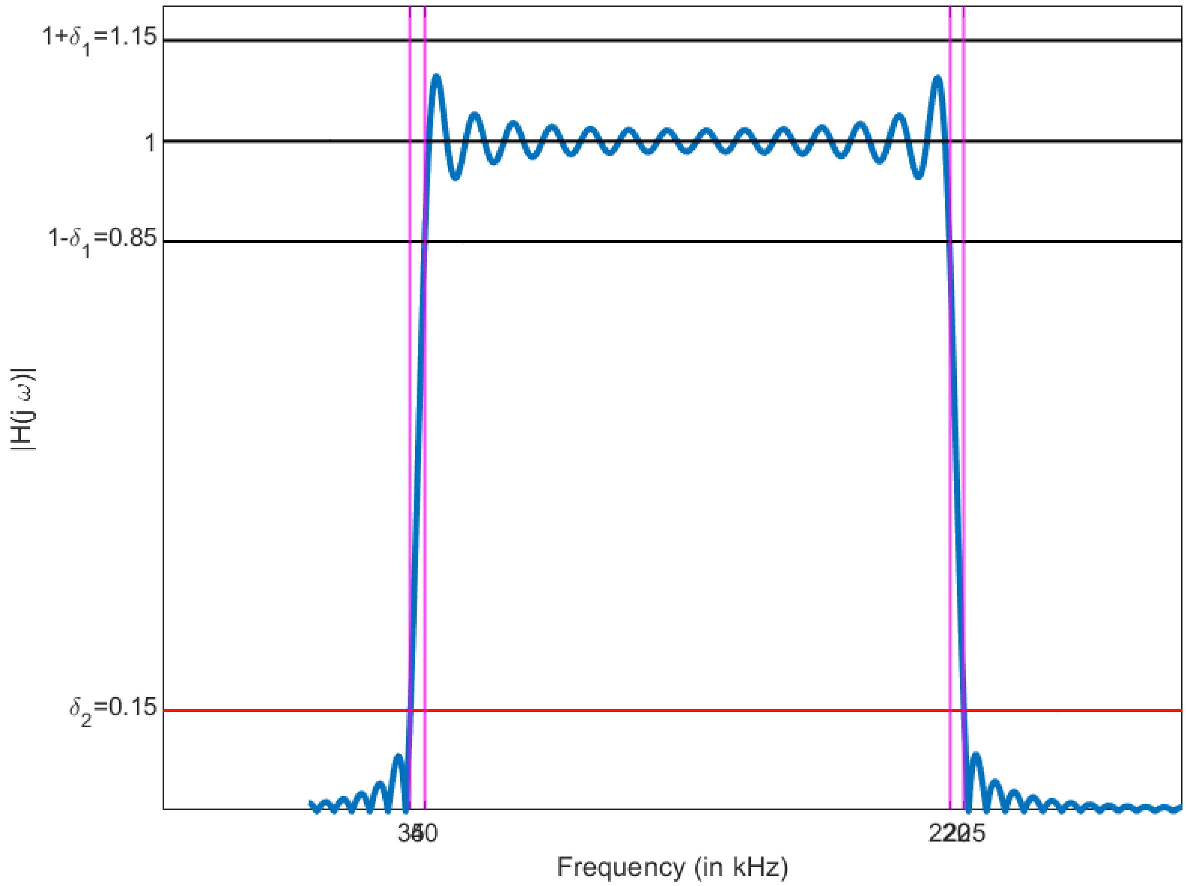

Figure 46: Bandpass Discrete-time Filter Response

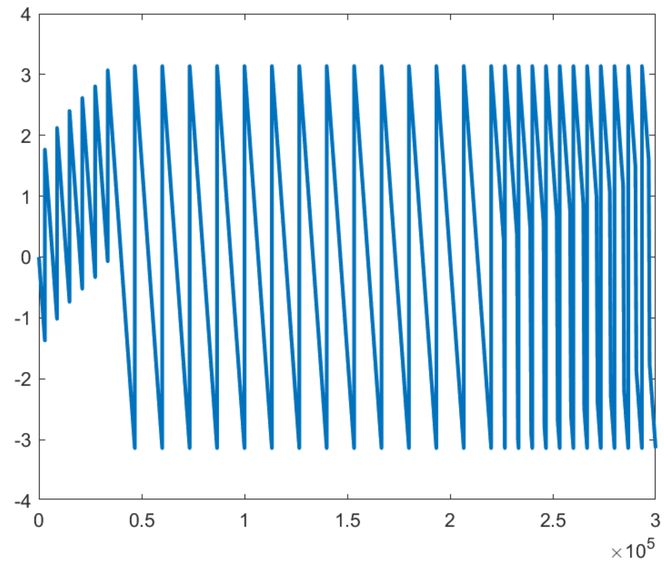

Figure 47: Bandpass Discrete-time Filter Response

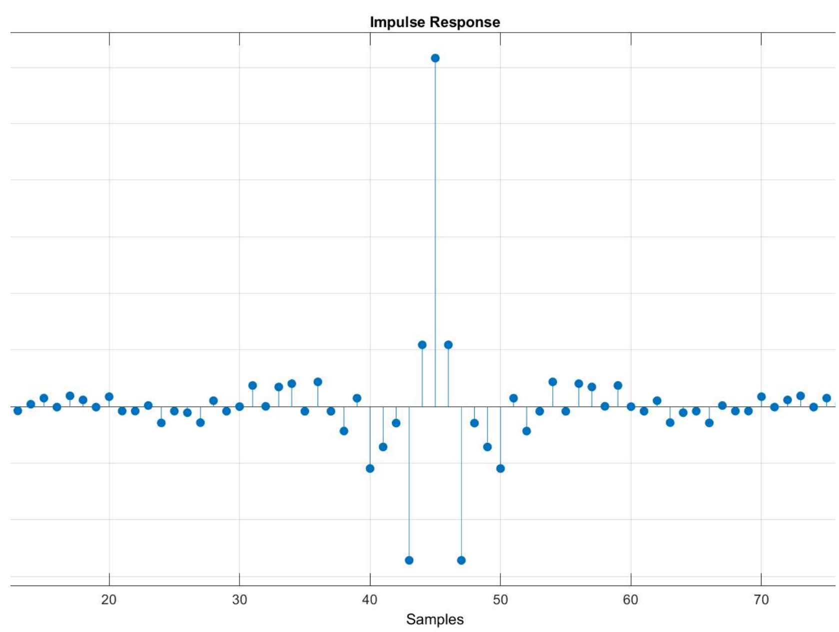

Figure 48: Bandpass Discrete-time Impulse Response

5.12 FIR Bandstop Filter

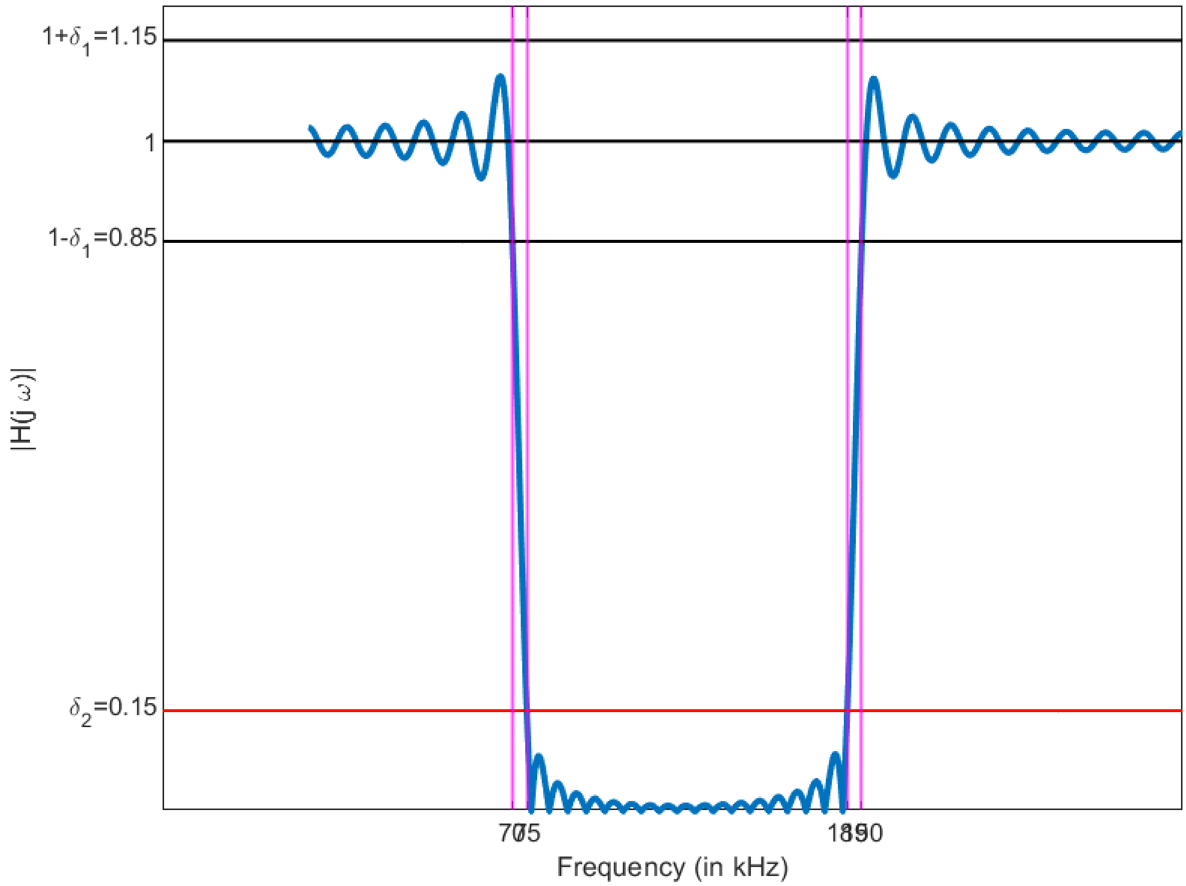

Figure 49: Bandstop Discrete-time Filter Response

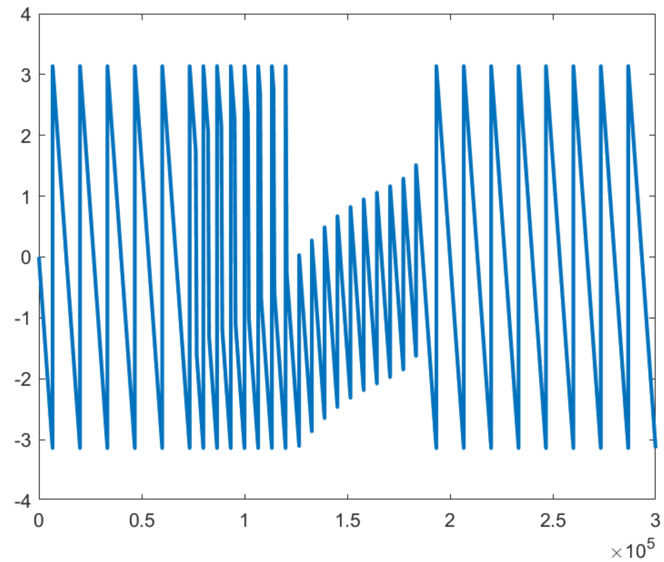

Figure 50: Bandstop Discrete-time Filter Response

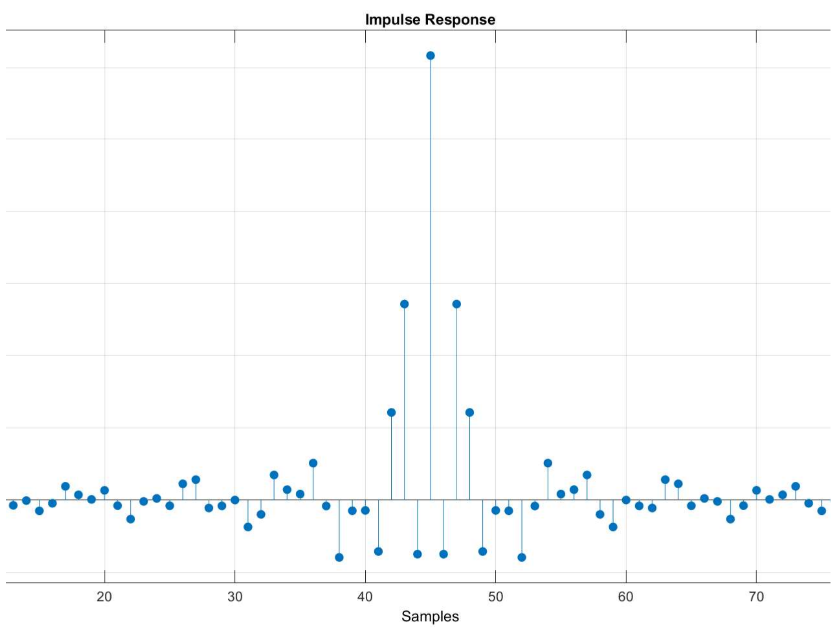

Figure 51: Bandstop Discrete-time Impulse Response

5.13 FIR Cascaded filter

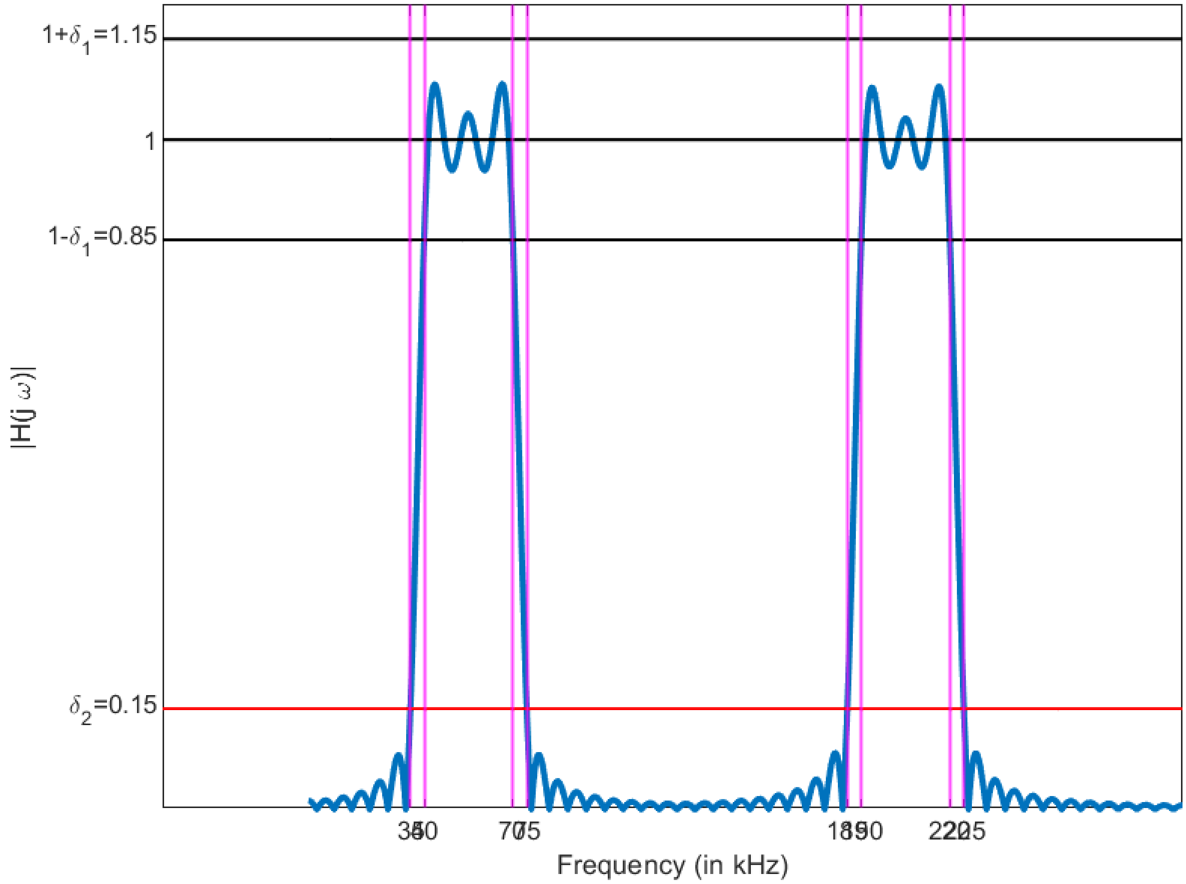

Figure 52: Cascade Discrete-time Filter Magnitude Response

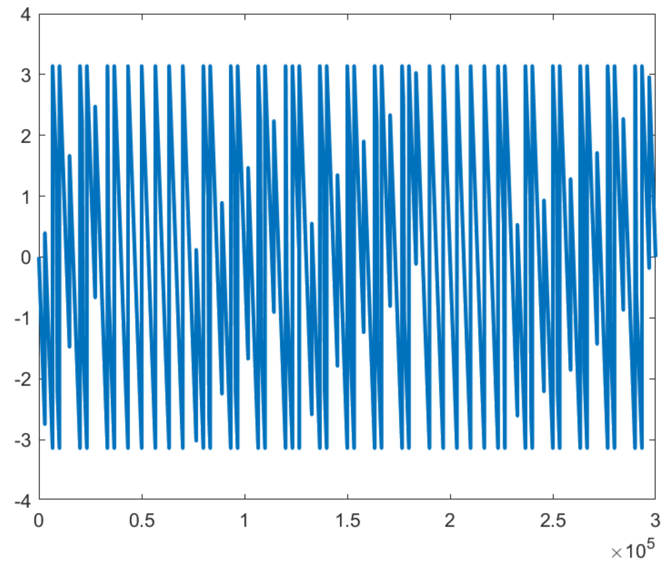

Figure 53: Cascade Discrete-time Filter Filter Response

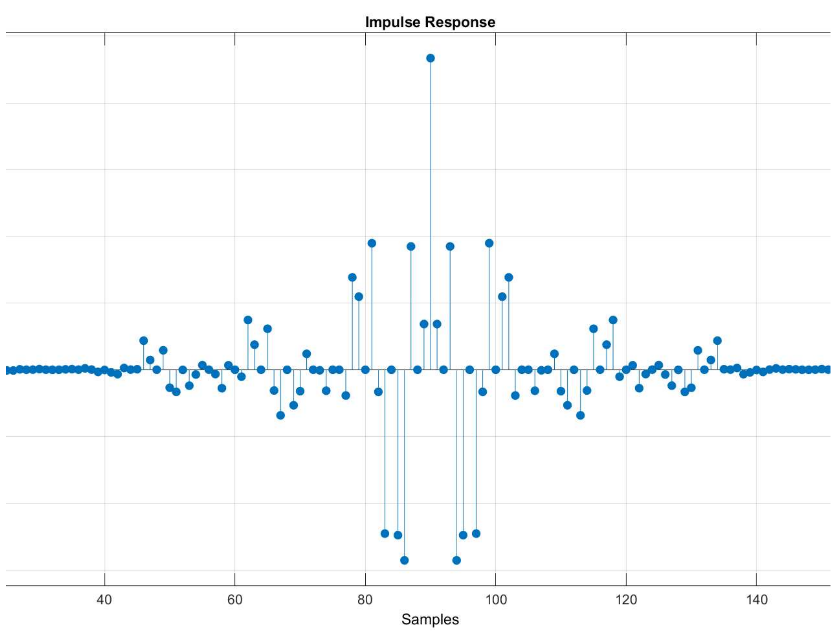

Figure 54: Cascade Discrete-time Impulse Response

6 Comparing the different filter types

- **Passband and stopband:** Butterworth filter has a monotonic response in both passband and stopband. Chebyshev filter has a monotonic stopband and an oscillatory passband response. Elliptical filters have an oscillatory response in both the passband and the stopband.

- **Transition Band:** Elliptical filter has the sharpest transition band. This reduces in the case of the Chebyshev filter and the Butterworth filter, and the FIR filter has the slowest passband to stopband transition.

- **Filter Order:** For the same specifications, elliptical filter has the smallest order (4), followed by Chebyshev (9), Butterworth (28), and FIR has the highest order (91).

- **Phase Response:** The FIR filters gives a perfectly linear phase response. Butterworth filter has an almost linear phase response. Chebyshev filter has a more non-linear phase response than Butterworth and the Elliptical filter has an even more non-linearity in its phase response.

7 Peer Reviews

7.1 Arya Vishe - 20d070018

I have thoroughly reviewed the filter design report of **Arya Vishe, 20d070018** and have found it to be correct. The filters were designed with proper steps, starting from the un-normalized specifications to the final discrete-time filter magnitude response. Sufficient simulation results and plots were provided for both the magnitude and phase response of the bandpass, bandstop, and multi-band filters.

7.2 Rajput Nikhileshsing Kailassing - 200070067

I have thoroughly reviewed the filter design report of **Rajput Nikhileshsing Kailassing, 200070067** and have found it to be correct. The filters were designed with proper steps, starting from the un-normalized specifications to the final discrete-time filter magnitude response. Sufficient simulation results and plots were provided for both the magnitude and phase response of the bandpass, bandstop, and multi-band filters.